2.7 Week 7 Tutorial: Diagonalization and Jordan normal form

HW: Let
$$A = \begin{pmatrix} 5 & -11 & 8 \\ 0 & 3 & -1 \\ -1 & 5 & -2 \end{pmatrix}$$

(a) Compute $A-2$, $(A-2)^2$ and $(A-2)^3$.

(b) Find a basis for $\ker(A-2)$.

(c) Find a basis for $\ker((A-2)^2)$.

(d) Find a basis for $\ker((A-2)^3)$.

Let $v_3 \in \ker((A-2)^3)$ such that $v_3 \notin \ker((A-2)^2)$. Let
$$v_2 = (A-2)v_3 \qquad \text{and} \qquad v_1 = (A-2)v_2.$$
Let P be the matrix that has v_1, v_2, v_3 as its columns. Show that
$$PAP^{-1} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$
A possible option for P is
$$P = \begin{pmatrix} 1 & 2 & -1 \\ 1 & -1 & 1 \\ 1 & -2 & 2 \end{pmatrix} \qquad \text{with} \qquad P^{-1} = \begin{pmatrix} 0 & 2 & -1 \\ 1 & -3 & 2 \\ 1 & -4 & 3 \end{pmatrix}$$

HW: Let
$$A = \begin{pmatrix} 3 & -3 & 2 \\ 1 & -1 & 2 \\ 1 & -3 & 4 \end{pmatrix}$$

(a) Compute $A-2$, $(A-2)^2$ and $(A-2)^3$.

(b) Find a basis for $\ker(A-2)$.

(c) Find a basis for $\ker((A-2)^2)$.

Find $v_2 \in \ker((A-2)^2)$ such that $v_2 \notin \ker(A-2)$. Let
$$v_1 = (A-2)v_2 \qquad \text{and choose a nonzero} \qquad v_3 \in \ker(A-2)$$
such that v_3 is not a multiple of v_1. Let P be the matrix that has v_1, v_2, v_3 as its columns. Check that
$$PAP^{-1} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$
A possible option for P is
$$P = \begin{pmatrix} 1 & 2 & -1 \\ 1 & -1 & 1 \\ 1 & -2 & 2 \end{pmatrix} \qquad \text{with} \qquad P^{-1} = \begin{pmatrix} 0 & 2 & -1 \\ 1 & -3 & 2 \\ 1 & -4 & 3 \end{pmatrix}$$

HW: Let
$$A = \begin{pmatrix} 6 & -15 & 11 \\ -2 & 11 & -7 \\ -5 & 21 & -14 \end{pmatrix}$$

(a) Compute $A-2$, $(A-2)^2$ and $(A-2)^2(A+1)$.

(b) Find a basis for $\ker(A-2)$.

(c) Find a basis for $\ker((A-2)^2)$.

(d) Find a basis for $\ker(A+1)$.

Find $v_2 \in \ker((A-2)^2)$ such that $v_2 \notin \ker(A-2)$. Let
$$v_1 = (A-2)v_2 \qquad \text{and choose a nonzero} \qquad v_3 \in \ker(A+1).$$
Let P be the matrix that has v_1, v_2, v_3 as its columns. Check that
$$PAP^{-1} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

HW: Let $A \in M_{3\times 3}(\mathbb{C})$. Assume that A has 3 linearly independent eigenvectors of eigenvalue 2. Prove that
$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$
Note that it follows that $A-2=0$.

HW: Let
$$A = \begin{pmatrix} 3 & -6 & 5 \\ -2 & 11 & -7 \\ -4 & 18 & -12 \end{pmatrix}$$

(a) Compute $A-2$, $(A-2)(A-1)$ and $(A-1)(A+1)$ and $(A-2)(A-1)(A+1)$.

(b) Find a basis for $\ker(A-2)$.

(c) Find a basis for $\ker(A-1)$.

(d) Find a basis for $\ker(A+1)$.

Let v_1, v_2, v_3 be nonzero vectors such that
$$v_1 \in \ker(A-2), \qquad v_2 \in \ker(A-1) \qquad \text{and} \qquad v_3 \in \ker(A+1).$$
Let P be the matrix that has v_1, v_2, v_3 as its columns. Check that
$$PAP^{-1} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$