1.8 Pop quiz on Lecture 8 material

1. Let $u = |1, 3, 1, 2\rangle$ and $v = |2, 1, -1, 3\rangle$ in $\mathbb{R}^4$. Determine $u - v$ and $d(u, v)$.

2. Let $u = |0, 2, 2, -1\rangle$ and $v = | -1, 1, 1, -1\rangle$ in $\mathbb{R}^4$. Determine $\langle u, v\rangle$ and $\|u\|$ and $\|v\|$ and check that the Cauchy-Schwarz inequality holds in this example.

3. Let $u = |2, -1, -2$ and $v = |2, 1, 3\rangle$ in $\mathbb{R}^3$. Find vectors v_1 and v_2 such that $v = v_1 + v_2$ and v_1 is parallel to u and v_2 is perpendicular to u.

4. Carefully prove that if $x, y \in \mathbb{R}^n$ then $\langle y, x\rangle = \langle x, y\rangle$.

5. Carefully prove that if $x, y, z \in \mathbb{R}^n$ then $\langle x, y + z\rangle = \langle x, y\rangle + \langle x, z\rangle$.

6. Carefully prove that if $x, y, z \in \mathbb{R}^n$ then $\langle x + y, z\rangle = \langle x, z\rangle + \langle y, z\rangle$.

7. Carefully prove that if x, y and $c \in \mathbb{R}$ then $\langle x, cy\rangle = c\langle x, y\rangle$.

8. Carefully prove that if x, y and $c \in \mathbb{R}$ then $\langle cx, y\rangle = c\langle x, y\rangle$.

9. Carefully prove that if x and $c \in \mathbb{R}$ then $\|cx\| = |c| \cdot \|x\|$.