Example M6 (topic 2 Example 6)

Find the inverse of $A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & 1 \\ 0 & 1 & 3 \end{pmatrix}$

Start with $AA^{-1} = 1$ which is

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & 1 \\ 0 & 1 & 3 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Left multiply both sides by

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ to get } \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 3 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Left multiply both sides by

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \text{ to get } \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix}$$

Left multiply both sides by

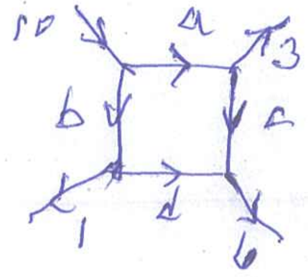
$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \text{ to get } \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} 2 & 1 & -1 \\ 3 & 3 & -2 \\ -1 & -1 & 1 \end{pmatrix}$$

Left multiply both sides by

$$\begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ to get } \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} -4 & -5 & -3 \\ 3 & 3 & -2 \\ -1 & -1 & 1 \end{pmatrix}$$

So $A^{-1} = \begin{pmatrix} -4 & -5 & -3 \\ 3 & 3 & -2 \\ -1 & -1 & 1 \end{pmatrix}$

Example L5 10 (Topic 1 Example 10) Linear Algebra Lect 5 A. Ram




has

Node 1: $10 = a + b$

Node 2: $a = 3 + c$

Node 3: $c + d = 6$

Node 4: $b = 1 + d$.

So

$$a + b + 0c + 0d = 0$$
$$a + 0b - c + 0d = 2$$
$$0a + 0b + c + d = 6$$
$$0a + b + 0c - d = 1$$

or $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 10 \\ 3 \\ 6 \\ 1 \end{pmatrix}$

Left multiply both sides by

$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ to get $\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \\ 6 \\ 1 \end{pmatrix}$

Left multiply both sides by

$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ to get $\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \\ 1 \\ 6 \end{pmatrix}$

Left multiply both sides by

$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ to get $\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 6 \\ 6 \end{pmatrix}$

Left multiply both sides by

$$\begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&0&1\\ 0&0&1&-1 \end{pmatrix} \text{ to get } \begin{pmatrix} 1&0&-1&0\\ 0&1&0&1\\ 0&0&1&1\\ 0&0&0&0 \end{pmatrix}\begin{pmatrix} a\\ b\\ c\\ d \end{pmatrix} = \begin{pmatrix} 3\\ 1\\ 6\\ 0 \end{pmatrix}$$

Left multiply both sides by

$$\begin{pmatrix} 1&0&1&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{pmatrix} \text{ to get } \begin{pmatrix} 1&0&0&0\\ 0&1&0&-1\\ 0&0&1&1\\ 0&0&0&0 \end{pmatrix}\begin{pmatrix} a\\ b\\ c\\ d \end{pmatrix} = \begin{pmatrix} 9\\ 1\\ 6\\ 0 \end{pmatrix}$$

This is the system

$$\begin{aligned} a &= 9\\ b-d &= 1\\ c+d &= 6\\ 0 &= 0 \end{aligned} \qquad \text{which is} \qquad \begin{aligned} a &= 9+0d\\ b &= 1+1d\\ c &= 6+(-1)d\\ d &= 0+1\cdot d \end{aligned}$$

where $d$ can be any number. So

$$Sol(A\vec{x}=\vec{b}) = \left\{ \begin{pmatrix} 9\\ 1\\ 6\\ 0 \end{pmatrix} + d\begin{pmatrix} 0\\ +1\\ -1\\ 1 \end{pmatrix} \;\middle|\; d\in\mathbb{Q} \right\}$$

$$= \begin{pmatrix} 9\\ 1\\ 6\\ 0 \end{pmatrix} + \mathbb{Q}\text{-span}\left\{ \begin{pmatrix} 0\\ 1\\ -1\\ 1 \end{pmatrix} \right\}$$

**Transpose** Let $t, s \in \mathbb{Z}_{>0}$ and $A \in M_{t\times s}(\mathbb{Q})$.

The $(i,j)$ entry of $A^T$ is

the $(j,i)$ entry of $A$.

**Example M4** (Topic 2 Example 4)

If $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ then $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$

**Trace** Let $n \in \mathbb{Z}_{>0}$ and $A \in M_{n\times n}(\mathbb{Q})$.

The **trace** of $A$ is

the sum of the diagonal entries.

**Example M3** Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$

$$\mathrm{Tr}(A) = \mathrm{Tr}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = 1+5+9 = 15.$$