1.24 Pop quiz on Lecture 24 material

1. Let $D = \begin{pmatrix} -4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. Compute D^{10}.

2. Assume that $D, P \in M_{n\times n}(\mathbb{Q})$ and P is invertible. Let $A = PDP^{-1}$ and let $k \in \mathbb{Z}_{>0}$. Show that $A^k = PD^kP^{-1}$.

3. Let
$$A = \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix}, \quad P = \begin{pmatrix} -2 & 2 \\ 1 & 1 \end{pmatrix}.$$
Show that $A = PDP^{-1}$ and compute A^{10}.

4. Let $r_0, p_0, w_0 \in \mathbb{R}$ and let
$$x_0 = \begin{pmatrix} r_0 \\ p_0 \\ w_0 \end{pmatrix} \qquad \text{and} \qquad T = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{2} \end{pmatrix}.$$
   (a) Find $P, D \in M_{3\times 3}(\mathbb{R})$ such that P is invertible, D is diagonal and $T = PDP^{-1}$.
   (b) Compute $\lim_{n\to\infty} T^n x_0$.