1.23 Pop quiz on Lecture 23 material

1. Let $\langle,\rangle\colon \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ be given by

$$\langle (u_1, u_2, u_3), (v_1, v_2, v_3)\rangle = u_1v_1 + 2u_2v_2 + u_3v_3.$$

Let $S = \{(1, 1, 1), (1, -1, 1), (1, 0, 1)\}$.

(a) Show that S is a orthogonal sequence in $\mathbb{R}^3$ with respect to $\langle,\rangle$.

(b) Make each element of S into a unit vector and show that this set $B = \{b_1, b_2, b_3\}$ of unit vectors is an orthonormal sequence in $\mathbb{R}^3$ with respect to $\langle,\rangle$.

(c) Let $x = |1, 1, -1\rangle$ and show that $x = \langle x, b_1\rangle b_1 + \langle x, b_2\rangle b_2 + \langle x, b_3\rangle b_3$.

2. Let $V = \mathbb{R}^3$ and let $W = \{|x, y, z\rangle \in \mathbb{R}^3 \mid x + y + z = 0\}$.

(a) Show that W is a subspace of $\mathbb{R}^3$.

(b) Let

$$b_1 = \tfrac{1}{\sqrt{2}}|1, -1, 0\rangle \quad \text{and} \quad b_2 = \tfrac{1}{\sqrt{6}}|1, 1, -2\rangle$$

and show that $\{b_1, b_2\rangle$ is an orthonormal basis of W with respect to the standard inner product on $\mathbb{R}^3$.

(c) Let $x = |1, 2, 3\rangle$. Compute $\mathrm{proj}_W(x)$.

(d) Let $x = |1, 2, 3\rangle$. Compute the shortest distance from x to W.

3. Let $V = \mathbb{R}^3$ with the standard inner product. Let $S = \{v_1, v_2, v_3\}$ with

$$v_1 = |1, 1, 1\rangle, \qquad v_2 = |0, 1, 1\rangle, \qquad v_3 = |0, 0, 1\rangle.$$

Use the Gram-Schmidt process of orthogonalization to convert S into an orthonormal basis B of V.

4. Let V be a $\mathbb{C}$-vector space with an inner product $\langle,\rangle\colon V \times V \to \mathbb{C}$. Show that if S is an orthonormal set then S is linearly independent.