1.22   Pop quiz on Lecture 22 material

1. Let V be an $\mathbb{C}$-vector space with an inner product $\langle,\rangle\colon V\times V\to\mathbb{C}$. Let $B=\{b_1,\ldots,b_n\{$ be a basis of V and let A be the Gram matrix of $\langle,\rangle$ with respect to the basis B. Let $u=u_1b_1+\cdots+u_nb_n\in V$ and let $v=v_1b_1+\cdots+v_nb_n\in V$. Prove that

$$\langle u,v\rangle=[u]_B^t A\overline{[v]_B}.$$

2. The $\mathbb{R}$-vector space $\mathbb{R}[x]_{\leq 2}$ has basis $B=\{1,x,x^2\}$.

   (a) Calculate the Gram matrix A of the standard inner product on $\mathbb{R}[]x]_{\leq 2}$ with respect to the basis B.

   (b) Let $u=7+3x+2x^2$ and $v=5+x^2$. Calculate

$$[u]_B\quad\text{and}\quad[v]_B\qquad\text{and verify that}\quad\langle u,v\rangle=[u]_B^t A[v]_B.$$