1.20 Pop quiz on Lecture 20 material

1. Determine (with proof) whether $A = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ is Hermitian.

2. Determine (with proof) whether $B = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}$ is Hermitian.

3. Determine (with proof) whether $U = \frac{1}{\sqrt{2}} \begin{pmatrix} -i & i \\ 1 & 1 \end{pmatrix}$ is unitary.

4. Determine (with proof) whether $Q = \begin{pmatrix} \cos(\theta) & -\sin(\theta)i \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$ is orthogonal.

5. Prove that if Q is an orthgonal matrix then $\det(Q)$ is either 1 or -1.

6. Prove that if $u, v \in \mathbb{R}^n$ and $\langle , \rangle$ is the standard inner product on $\mathbb{R}^n$ and Q is an $n \times n$ orthgonal matrix then $\langle Qu, Qv \rangle = \langle u, v \rangle$.

7. Let $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$.

   (a) Show that A is a symmetric matrix.

   (b) Find an orthgonal matrix Q and a diagonal matrix D such that $A = QDQ^T$.

8. Let $A = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$.

   (a) Show that A is a Hermitian matrix.

   (b) Find an unitary matrix U and a diagonal matrix D such that $A = UD\overline{U}^T$.