1.19 Pop quiz on Lecture 19 material

1. Let $A = \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix}$.
   (a) Find the eigenvalues and eigenvectors of A as an element of $M_{2\times 2}(\mathbb{C})$.
   (b) Find $P \in GL_2(\mathbb{Q})$ and a diagonal matrix D such that $PAP^{-1} = D$.
   (c) Find the characteristic polynomial of A and factor it into linear factors.

2. Let $A = \begin{pmatrix} 2 & -3 & 6 \\ 0 & 5 & -6 \\ 0 & 1 & 0 \end{pmatrix}$.
   (a) Find the eigenvalues and eigenvectors of A as an element of $M_{3\times 3}(\mathbb{C})$.
   (b) Find $P \in GL_2(\mathbb{Q})$ and a diagonal matrix D such that $PAP^{-1} = D$.
   (c) Find the characteristic polynomial of A and factor it into linear factors.

3. Let $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.
   (a) Find the eigenvalues and eigenvectors of A as an element of $M_{2\times 2}(\mathbb{R})$.
   (a) Find the eigenvalues and eigenvectors of A as an element of $M_{2\times 2}(\mathbb{R})$.
   (b) Find $P \in GL_2(\mathbb{C})$ and a diagonal matrix D such that $PAP^{-1} = D$.
   (c) Find the characteristic polynomial of A and factor it into linear factors.

4. Give an example of a 2×2 matrix A that does not have 2 linearly independent eigenvectors as an element of $M_{2\times 2}(\mathbb{C}$. Be sure you *prove* that the matrix A that you give does not have 2 linearly independent eigenvectors.

5. Give an example of a 2×2 matrix A that is not diagonalizable as an element of $M_{2\times 2}(\mathbb{C}$. Be sure you *prove* that the matrix A that you give is not diagonalizatble.