1.17 Pop quiz on Lecture 17 material

1. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which is reflection across the y-axis. Fix the unit square to be the square with vertices $|0,0\rangle$, $|1,0\rangle$, $|1,1\rangle$, $|0,1\rangle$.
   (a) Draw a picture illustrating the image of the unit square under the transformation T.
   (b) Determine (with proof) the matrix of the linear transformation T with respect to the basis $S = \{|1,0\rangle, |0,1\rangle\}$ of $\mathbb{R}^2$.
2. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which is projection onto the x axis. Fix the unit square to be the square with vertices $|0,0\rangle$, $|1,0\rangle$, $|1,1\rangle$, $|0,1\rangle$.
   (a) Draw a picture illustrating the image of the unit square under the transformation T.
   (b) Determine (with proof) the matrix of the linear transformation T with respect to the basis $S = \{|1,0\rangle, |0,1\rangle\}$ of $\mathbb{R}^2$.
3. Let $c \in \mathbb{R}_{>0}$ and let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which is the shear by a factor of c along the x-axis. Fix the unit square to be the square with vertices $|0,0\rangle$, $|1,0\rangle$, $|1,1\rangle$, $|0,1\rangle$.
   (a) Draw a picture illustrating the image of the unit square under the transformation T.
   (b) Determine (with proof) the matrix of the linear transformation T with respect to the basis $S = \{|1,0\rangle, |0,1\rangle\}$ of $\mathbb{R}^2$.
4. Let $c \in \mathbb{R}_{>0}$ and let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which is the shear by a factor of c along the x-axis followed by a reflection across the y-axis. Fix the unit square to be the square with vertices $|0,0\rangle$, $|1,0\rangle$, $|1,1\rangle$, $|0,1\rangle$.
   (a) Draw a picture illustrating the image of the unit square under the transformation T.
   (b) Determine (with proof) the matrix of the linear transformation T with respect to the basis $S = \{|1,0\rangle, |0,1\rangle\}$ of $\mathbb{R}^2$.
5. Let $c \in \mathbb{R}_{>0}$ and let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which is stretching of the x-axis by a factor of c. Fix the unit square to be the square with vertices $|0,0\rangle$, $|1,0\rangle$, $|1,1\rangle$, $|0,1\rangle$.
   (a) Draw a picture illustrating the image of the unit square under the transformation T.
   (b) Determine (with proof) the matrix of the linear transformation T with respect to the basis $S = \{|1,0\rangle, |0,1\rangle\}$ of $\mathbb{R}^2$.
6. Let $\theta \in \mathbb{R}_{[0,2\pi)}$ and let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which is rotation by θ (about the origin counterclockwise). Fix the unit square to be the square with vertices $|0,0\rangle$, $|1,0\rangle$, $|1,1\rangle$, $|0,1\rangle$.
   (a) Draw a picture illustrating the image of the unit square under the transformation T.
   (b) Determine (with proof) the matrix of the linear transformation T with respect to the basis $S = \{|1,0\rangle, |0,1\rangle\}$ of $\mathbb{R}^2$.
7. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which is reflection in the line $y = 5x$. Fix the unit square to be the square with vertices $|0,0\rangle$, $|1,0\rangle$, $|1,1\rangle$, $|0,1\rangle$.
   (a) Draw a picture illustrating the image of the unit square under the transformation T.
   (b) Determine (with proof) the matrix of the linear transformation T with respect to the basis $S = \{|1,0\rangle, |0,1\rangle\}$ of $\mathbb{R}^2$.

8. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by

$$T(|x, y\rangle) = |3x - y, -x + 3y\rangle.$$

Fix the unit square to be the square with vertices $|0, 0\rangle$, $|1, 0\rangle$, $|1, 1\rangle$, $|0, 1\rangle$.

(a) Draw a picture illustrating the image of the unit square under the transformation T.

(b) Determine (with proof) the matrix X of the linear transformation T with respect to the basis $S = \{|1, 0\rangle, |0, 1\rangle\}$ of $\mathbb{R}^2$.

(c) Determine (with proof) the transition matrix P between the basis S and the basis $B = \{|1, 1\rangle, |-1, 1\rangle\}$.

(d) Determine (with proof) the transition matrix Q between the basis B and the basis S.

(e) Check that P is the inverse of Q.

(f) Determine (with proof) the matrix Y of the linear transformation T with respect to the basis B.

(g) Check that $Y = PXP^{-1}$.