1.16 Pop quiz on Lecture 16 material

1. (a) (a)] Find the coordinate vector of $v = |1,5\rangle$ with respect to the basis $S = \{|1,0\rangle, |0,1\rangle\}$ of $\mathbb{R}^2$.

   (b) (b)] Find the coordinate vector of $v = |1,5\rangle$ with respect to the basis $B = \{|2,1\rangle, |-1,1\rangle\}$ of $\mathbb{R}^2$.

2. (a) (a)] Find the coordinate vector of $p = 1 + 7x - 9x^2$ with respect to the basis $S = \{1, x, x^2\}$ of $\mathbb{R}[x]_{\le 2}$.

   (b) (b)] Find the coordinate vector of $p = 1 + 7x - 9x^2$ with respect to the basis $B = \{2, \frac{1}{2}x, -3x^2\}$ of $\mathbb{R}[x]_{\le 2}$.

3. Let $T\colon U \to V$ and $f\colon V \to W$ be linear transformations. Let

$$B \text{ be a basis of } U, \quad C \text{ a basis of } V, \quad D \text{ a basis of } W.$$

   Show that

$$[f \circ T]_{DB} = [f]_{DC}[T]_{CB}.$$

4. Let V be a vector space. Let

$$S = \{v_1, v_2, v_3, v_4\} \text{ be a basis of } V \quad \text{and} \quad B = \{b_1, b_2, b_3\} \text{ be a another basis of } V.$$

   Show that

$$[I]_{SS} = 1, \quad [I]_{BB} = 1, \quad [I]_{SB}[I]_{BS} = [I]_{SS}, \quad \text{and} \quad [I]_{BS}[I]_{SB} = [I]_{BB}.$$

5. Let $T\colon \mathbb{R}[s]_{\le 3} \to \mathbb{R}[x]_{\le 2}$ be the linear transformation given by

$$T(p) = \frac{dp}{dx}.$$

   Find the matrix of T with respect to the basis $S = \{1, x, x^2, x^3\}$ of $\mathbb{R}[x]_{\le 3}$ and the basis $B = \{1, x, x^2\}$ of $\mathbb{R}[x]_{\le 2}$.

6. Let $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ be the function given by

$$T(|x_1, x_2, x_3\rangle) = |x_2 - 2x_3, 3x_1 + x_3\rangle.$$

   Find the matrix of T with respect to the basis $B = \{|1,0,0\rangle, |0,1,0\rangle, |0,0,1\rangle\}$ of $\mathbb{R}^3$ and the basis $S = \{|1,0\rangle, |0,1\rangle\}$ of $\mathbb{R}^2$.

7. Let $T\colon M_{2\times 2}(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ be the linear transformation given by

$$T(Q) = Q^t, \quad \text{where } Q^t \text{ is the transpose of the matrix } Q.$$

   find the matrix of T with respect to the basis $B = \{E_{11}, E_{12}, E_{21}, E_{22}\}$ of $M_{2\times 2}(\mathbb{R})$ (where E_{ij} has 1 in the (i,j) entry and 0 elsewhere).

8. Let $T\colon \mathbb{R}[x]_{\le 2} \to \mathbb{R}[x]_{\le 1}$ be the linear transformation given by

$$T(a_0 + a_1x + a_2x^2) = (a_0 + a_2) + a_0x.$$

(a) Find the matrix of T with respect to the basis $B = \{1, x, x^2\}$ of $\mathbb{R}[x]_{\le 2}$ and the basis $C = \{1, x\}$ of $\mathbb{R}[x]_{\le 1}$.

(b) Find the matrix of T with respect to the basis $B = \{1, x, x^2\}$ of $\mathbb{R}[x]_{\le 2}$ and the basis $D = \{2, 3x\}$ of $\mathbb{R}[x]_{\le 1}$.

9. Suppose that $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation and that the matrix of T with respect to

$$\text{the basis } A = \{(1,0,0),(0,1,0),(0,0,1)\} \text{ of } \mathbb{R}^3 \quad \text{and} \quad \text{the basis } S = \{(1,0),(0,1)\} \text{ of } \mathbb{R}^2$$

is

$$[T]_{SA} = \begin{pmatrix} 5 & 1 & 0 \\ 1 & 5 & -2 \end{pmatrix}.$$

Find the matrix of T with respect to

$$\text{the basis } B = \{(1,1,0),(1,-1,0),(1,-1,-2)\} \text{ of } \mathbb{R}^3 \quad \text{and} \quad \text{the basis } C = \{(1,1),(1,-1)\} \text{ of } \mathbb{R}^2.$$