1.15 Pop quiz on Lecture 15 material

1. Let $T\colon V \to W$ be an $\mathbb{R}$-linear transformation.

   Show that $\ker(T)$ is a subspace of V.

2. Let $T\colon V \to W$ be an $\mathbb{R}$-linear transformation.

   Show that $\mathrm{im}(T)$ is a subspace of V.

3. Let $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation given by

$$T(x, y, x) = (2x - y, y + z).$$

   Find bases for $\ker(T)$ and $\mathrm{im}(T)$ and verify the rank nullity theorem.

4. Let $\mathbb{R}[x]_{\le 2} = \{a_0 + a_1x + a_2x^2 \mid a_0, a_1, a_2 \in \mathbb{R}\}$ and let $\mathbb{R}[x]_{\le 1} = \{a_0 + a_1x \mid a_0, a_1 \in \mathbb{R}\}$. Let $T\colon \mathbb{R}[x]_{\le 2} \to \mathbb{R}[x]_{\le 1}$ be the linear transformation given by

$$T(a_0 + a_1x + a_2x^2) = (a_0 - a_1 + a_2)(1 + 2x).$$

   (a) Find bases for $\ker(T)$ and $\mathrm{im}(T)$.
   (b) Is T injective?
   (c) Is T surjective?