1.14 Pop quiz on Lecture 14 material

1. Let V be a $\mathbb{Q}$-vector space and let $v_1, v_2, v_3, v_4, v_5 \in V$. Let $S = \{v_1, v_2, v_3, v_4, v_5\}$

$$\text{Show that } \mathbb{Q}\text{-span}(S) \text{ is a subspace of } V.$$

2. Let $\mathbb{R}[x]_{\le 2}$ be the $\mathbb{R}$-vector space of polynomials of degree ≤ 2 with coefficients in $\mathbb{R}$.

$$\text{Is} \quad 1 - 2x - x^2 \in \mathbb{R}\text{-span}\{1 + x + x^2, 3 + x^2\}?$$

3. Let S be the subset of $\mathbb{R}^3$ given by

$$S = \{|1,1,1\rangle, |2,2,2\rangle, |3,3,3\rangle\}. \qquad \text{Determine } \mathbb{R}\text{-span}(S).$$

4. Let $\mathbb{R}[x]_{\le 2}$ be the $\mathbb{R}$-vector space of polynomials of degree ≤ 2 with coefficients in $\mathbb{R}$. Let S be the subset of $\mathbb{R}[x]_{\le 2}$ given by

$$S = \{1 + 2x, 1 + 5x + 3x^2, x + x^2\}. \quad \text{Show that } \mathbb{R}\text{-span}(S) = \mathbb{R}[x]_{\le 2}.$$

5. If $S = \{(1,-1), (2,4)\}$ a basis of $\mathbb{R}^2$.

6. Let S be a subset of $M_{2\times 2}(\mathbb{R})$ given by

$$S = \left\{ \begin{pmatrix} 1 & 3 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} -2 & 1 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 10 \\ 4 & 2 \end{pmatrix} \right\}. \qquad \text{is } S \text{ linearly independent?}$$