1.13 Pop quiz on Lecture 13 material

1. Let $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix}$. Show that the function $T\colon \mathbb{R}^4 \to \mathbb{R}^2$ given by

$$T(x) = Ax \quad \text{is a linear tranformation.}$$

2. Let $t, s \in \mathbb{Z}_{>0}$ and let $A \in M_{t\times s}(\mathbb{R})$. Show that the function $T\colon \mathbb{R}^s \to \mathbb{R}^t$ given by

$$T(x) = Ax \quad \text{is a linear tranformation.}$$

3. Let $n \in \mathbb{Z}_{>0}$. Show that the function $T\colon M_{n\times n}(\mathbb{Q}) \to \mathbb{Q})$ given by

$$T(A) = \mathrm{tr}(A) \quad \text{is a linear tranformation.}$$

4. Show that the function $T\colon M_{2\times 2}(\mathbb{Q}) \to \mathbb{Q}$ given by

$$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc \quad \text{is a linear tranformation.}$$

5. Let $n \in \mathbb{Z}_{>0}$. Show that the function $T\colon M_{n\times n}(\mathbb{Q}) \to \mathbb{Q})$ given by

$$T(A) = \det(A) \quad \text{is a linear tranformation.}$$