$$= \int \frac{1}{2} \cdot \frac{1}{(x+1)^2+1} \cdot 2(x+1)\, dx - 3 \int \frac{1}{(x+1)^2+1}\, dx$$

$= \frac{1}{2} \log((x+1)^2+1) - 3 \arctan(x+1) + c$, where c is a constant.

<u>Problem</u> $\displaystyle\int \frac{9x+1}{(x-3)(x+1)}\, dx$

$$= \int \frac{9(x+1)-8}{(x-3)(x+1)}\, dx = \int \frac{9(x+1)}{(x-3)(x+1)}\, dx - 8 \int \frac{1}{(x-3)(x+1)}\, dx.$$

$$= 9 \int \frac{1}{x-3}\, dx - 8 \int \frac{1}{-4} \cdot \frac{(x-3)-(x+1)}{(x-3)(x+1)}\, dx.$$

$$= 9 \int \frac{1}{x-3}\, dx + 2 \left(\int \frac{x-3}{(x-3)(x+1)}\, dx - \int \frac{(x+1)}{(x-3)(x+1)}\, dx \right)$$

$$= 9 \int \frac{1}{x-3}\, dx + 2 \int \frac{1}{x+1}\, dx - 2 \int \frac{1}{x-3}\, dx$$

$$= 7 \int \frac{1}{x-3}\, dx + 2 \int \frac{1}{x+1}\, dx$$

$= 7 \log(x-3) + 2 \log(x+1) + c$, where c is a constant.

30.04.2015

Integrals

(1)

Integration by parts 3.22

(1)

$$\int x\cos(x)\,dx = \int\left(x\cos(x) + \sin(x) - \sin(x)\right) dx$$

$$= \int\left(\frac{d(x\sin(x))}{dx} - \sin(x)\right) dx$$

$$= x\sin(x) + \cos(x) + c,$$ where c is a constant.

(2)

$$\int x^2 e^x\,dx = \int\left(x^2e^x + 2xe^x - 2xe^x\right) dx$$

$$= \int \frac{d(x^2e^x)}{dx}\,dx - 2\int xe^x\,dx$$

$$= \int \frac{d(x^2e^x)}{dx}\,dx - 2\int\left(xe^x + e^x - e^x\right) dx$$

$$= \int \frac{d(x^2e^x)}{dx}\,dx - 2\left(\int \frac{d(xe^x)}{dx}\,dx - \int e^x\,dx\right)$$

$$= \int \frac{d(x^2e^x)}{dx}\,dx - 2\int \frac{d(xe^x)}{dx}\,dx + 2\int e^x\,dx$$

$$= x^2e^x - 2xe^x + 2e^x + c,$$ where c is a constant.

(3) $\int \log(x)\,dx = \int \left(\log(x)\cdot 1 + \frac{1}{x}\cdot x - \frac{1}{x}x\right)dx$

$= \int \left(\frac{d(\log(x)\cdot x)}{dx} - 1\right)dx = \log(x)\cdot x - x + c$

$= x\log(x) - x + c$, where c is a constant.

(4) $\int xe^x\,dx = \int (xe^x + e^x - e^x)\,dx$

$= \int \left(\frac{d(xe^x)}{dx} - e^x\right)dx = xe^x - e^x + c$, where c is a constant.

(5) $\int x\log(x)\,dx = \frac{1}{2}\int 2x\log(x)\,dx$

$= \frac{1}{2}\left(\int 2x\log(x) + x^2\cdot\frac{1}{x} - x^2\cdot\frac{1}{x}\right)dx$

$= \frac{1}{2}\int \left(\frac{d(x^2\log(x))}{dx} - x\right)dx$

$= \frac{1}{2}\left(x^2\log(x) - \frac{1}{2}x^2\right) + c$

$= \frac{1}{2}x^2\log(x) - \frac{1}{4}x^2 + c$, where c is a constant.

30.04.2025 (3)

Integrals

(6) $\int e^x \sin(x)\,dx = \int \left(e^x \sin(x) + e^x \cos(x) - e^x \cos(x) \right) dx$

$= \int \left(\frac{d(e^x \sin(x))}{dx} - e^x \cos(x) \right) dx$

$= \int \frac{d(e^x \sin(x))}{dx}\,dx - \int \left(e^x \cos(x) + e^x \sin(x) + e^x \sin(x) \right) dx$

$= \int \frac{d(e^x \sin(x))}{dx}\,dx - \int \frac{d(e^x \cos(x))}{dx}\,dx - \int e^x \sin(x)\,dx.$

So

$$2\int e^x \sin(x)\,dx = \int \left(\frac{d(e^x \sin(x))}{dx} - \frac{d(e^x \cos(x))}{dx} \right) dx$$

$$= e^x \sin(x) - e^x \cos(x) + c.$$

So $\int e^x \sin(x)\,dx = \frac{1}{2} e^x \sin(x) - \frac{1}{2} e^x \cos(x) + c$,
where c is a constant.

(7) $\int \arcsin(2x)\,dx = \int \left(\arcsin(2x)\cdot 1 + \frac{2}{\sqrt{1-4x^2}}\cdot x - \frac{2}{\sqrt{1-4x^2}}\cdot x \right) dx$

$= \int \frac{d(\arcsin(x)\cdot x)}{dx} - 2\int \frac{1}{8} \frac{8x}{\sqrt{1-4x^2}}\,dx.$

$= x\arcsin(x) - \frac{1}{4}\cdot\frac{2}{3}(1-4x^2)^{3/2} + c = x\arcsin(x) - \frac{1}{6}(1-4x^2)^{3/2} + c$

where c is a constant

(8) $\int x^3\sqrt{4-x^2}\,dx = \int -\frac{1}{2}x^2\,(4-x^2)^{\frac{1}{2}}(-2x)\,dx.$

$= -\frac{1}{2}\cdot\frac{2}{3}\int\left(\frac{3}{2}x^2(-2x)(4-x^2)^{\frac{1}{2}} + 2x(4-x^2)^{3/2} - 2x(4-x^2)^{3/2}\right)dx$

$= -\frac{1}{3}\int\left(\frac{d\left(x^2(4-x^2)^{3/2}\right)}{dx} - 2x(4-x^2)^{3/2}\right)dx.$

$= -\frac{1}{3}\left(x^2(4-x^2)^{3/2} - \frac{2}{5}(4-x^2)^{5/2}\right) + c.$

$= -\frac{1}{3}x^2(4-x^2)^{3/2} + \frac{2}{15}(4-x^2)^{5/2} + c$, where c is a constant.

(9) $\int \arctan(x)\,dx = \int\left(\arctan(x)\cdot 1 + \frac{1}{x^2+1}\cdot x - \frac{1}{x^2+1}\cdot x\right)dx$

$= \int\left(\frac{d\left(x\arctan(x)\right)}{dx} - \frac{1}{2}\frac{1}{(x^2+1)}2x\right)dx$

$= x\arctan(x) - \frac{1}{2}\log(x^2+1) + c$, where c is a constant.

(10) $\int x\arctan(x)\,dx = \int \frac{1}{2}\arctan(x)\,2x\,dx$

$= \frac{1}{2}\int\left((2x)\arctan(x) + x^2\cdot\frac{1}{x^2+1}\right) - \frac{x^2}{x^2+1}\Big)\,dx$

$= \frac{1}{2}\int \frac{d\left(x^2\arctan(x)\right)}{dx} - \frac{x^2+1-1}{x^2+1}\Big)\,dx$

30.04.2026 Integrals (5)

$$= \frac{1}{2}\int\left(\frac{d(x^2\arctan(x))}{dx} - 1 + \frac{1}{x^2+1}\right)dx$$

$$= \frac{1}{2}\left(x^2\arctan(x) - x + \arctan(x)\right) + c$$

$= \frac{1}{2}x^2\arctan(x) - \frac{1}{2}x + \frac{1}{2}\arctan(x) + c$, where c is a constant.

3.33 (11)

$$\int \frac{7x-5}{(x-2)(x+1)}dx = \int \frac{7(x-2)+9}{(x-2)(x+1)}dx$$

$$= \int\left(\frac{7}{x+1} + 9\,\frac{1}{(x-2)(x+1)}\right)dx$$

$$= \int\left(\frac{7}{x+1} + \frac{9}{-3}\,\frac{((x-2)-(x+1))}{(x-2)(x+1)}\right)dx$$

$$= \int\left(\frac{7}{x+1} - 3\,\frac{1}{x+1} + 3\,\frac{1}{x-2}\right)dx$$

$= 4\log(x+1) + 3\log(x-2) + c$, where c is a constant.

3.35 (12)

$$\int \frac{2x+1}{(x+1)^2}dx = \int \frac{2(x+1) - 1}{(x+1)^2}dx = \int\left(\frac{2}{x+1} - (x+1)^{-2}\right)dx$$

$= 2\log(x+1) + (x+1)^{-1} + c$, where c is a constant.

30.04.2016
Integrals
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(13) $\displaystyle\int \frac{x^3+2x-1}{x^2-1}\,dx = \int \frac{x(x^2-1)+3x-1}{x^2-1}\,dx$

$\displaystyle = \int \left(x + \frac{3}{2}\cdot\frac{2x}{x^2-1} - \frac{1}{x^2-1} \right) dx$

$\displaystyle = \int \left(x + \frac{3}{2}\,\frac{2x}{x^2-1} - \frac{1}{(x+1)(x-1)} \right) dx$

$\displaystyle = \int \left(x + \frac{3}{2}\cdot\frac{2x}{x^2-1} - \frac{1}{2}\cdot\frac{x+1-(x-1)}{(x+1)(x-1)} \right) dx$

$\displaystyle = \int \left(x + \frac{3}{2}\cdot\frac{2x}{x^2-1} - \frac{1}{2}\,\frac{1}{x-1} + \frac{1}{2}\,\frac{1}{x+1} \right) dx$

$\displaystyle = \frac{1}{2}x^2 + \frac{3}{2}\log(x^2-1) - \frac{1}{2}\log(x-1) + \frac{1}{2}\log(x-1) + c,$
where c is a constant

(14) $\displaystyle\int \frac{x^2+5x+3}{x^3+x}\,dx = \int \frac{x^2+5x+3}{x(x^2+1)}\,dx$

$\displaystyle = \int \frac{x^2+1+5x+2}{x(x^2+1)}\,dx = \int \left(\frac{1}{x} + \frac{5}{x^2+1} + \frac{2}{(x^2+1)x} \right) dx$

$\displaystyle = \int \frac{1}{x} + \frac{5}{x^2+1} - 2\,\frac{x\cdot x-(x^2+1)}{x(x^2+1)}\,dx$

$\displaystyle = \int \left(\frac{1}{x} + \frac{5}{x^2+1} - 2\left(\frac{x}{x^2+1} - \frac{1}{x} \right) \right) dx = \int \left(\frac{1}{x} + \frac{5}{x^2+1} - \frac{2x}{x^2+1} + \frac{2}{x} \right) dx$

29.04.2026 5
Calculussheet21
A.Rem

Problem $\displaystyle\int \frac{3x^2-2x+1}{(x+1)(x^2+2x+2)}\,dx$

$$= \int \frac{3(x^2+2x+2)-8x-5}{(x+1)(x^2+2x+2)}\,dx$$

$$= \int \frac{3}{x+1}\,dx - \int \frac{8x+5}{(x+1)(x^2+2x+2)}\,dx$$

$$= \int \frac{3}{x+1}\,dx - \int \frac{8(x+1)-3}{(x+1)(x^2+2x+2)}\,dx$$

$$= 3\int \frac{1}{x+1}\,dx - 8\int \frac{1}{x^2+2x+2}\,dx - 3\int \frac{1}{(x+1)(x^2+2x+2)}\,dx$$

$$= 3\int \frac{1}{x+1}\,dx - 8\int \frac{1}{(x+1)^2+1}\,dx - 3\int \frac{(x^2+2x+2)-(x+1)(x+1)}{(x+1)(x^2+2x+2)}\,dx$$

$$= 3\int \frac{1}{x+1}\,dx - 8\int \frac{1}{x^2+2x+2}\,dx - 3\int \left(\frac{1}{x+1} - \frac{x+1}{x^2+2x+2}\right)dx.$$

$$= -8\int \frac{1}{(x+1)^2+1}\,dx + 3\int \frac{1}{2}\cdot\frac{1}{(x+1)^2+1}\cdot 2(x+1)\,dx$$

$$= -8\arctan(x+1) + \frac{3}{2}\log((x+1)^2+1) + c,$$

where c is a constant.