27.04.2024
Calculus Lect 24 (1)
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Let $J = \sin(x)$

Then $\frac{dJ}{dx} = \cos(x)$.

<u>Problem</u> $\int \sin(x)^{25} \cos(x)^5 \, dx$

$= \int \sin(x)^{25} \cos(x)^4 \cos(x) \, dx$

$= \int \sin(x)^{25} (\cos(x)^2)^2 \cos(x) \, dx$

$= \int \sin(x)^{25} ((1-\sin(x))^2)^2 \cos(x) \, dx$

$= \int \sin(x)^{25} (1 - 2\sin(x)^2 + \sin(x)^4) \cos(x) \, dx$

$= \int (\sin(x)^{25} - 2\sin(x)^{27} + \sin(x)^{29}) \cos(x) \, dx.$

$= \int (\sin(x)^{25} \cos(x) - 2\sin(x)^{27} \cos(x) + \sin(x)^{29} \cos(x)) \, dx$

$= \frac{1}{26} \sin(x)^{26} - 2 \cdot \frac{1}{28} \sin(x)^{28} + \frac{1}{30} \sin(x)^{30} + c,$

where c is a constant.

<u>Problem</u> $\int x e^{6x} \, dx = \int \frac{1}{6} x 6 e^{6x} \, dx$

$= \frac{1}{6} \int (x 6e^{6x} + 1 \cdot e^{6x} - e^{6x}) \, dx$

$= \frac{1}{6} \int \left(\frac{d(xe^{6x})}{dx} - e^{6x} \right) dx$

$= \frac{1}{6} \left(xe^{6x} - \frac{1}{6} e^{6x} \right) + c$

Let $F = x$ and $S = e^{6x}$.
Then $\frac{dF}{dx} = 1$ and $\frac{dS}{dx} = 6e^{6x}$.

$= \frac{1}{6} xe^{6x} - \frac{1}{36} e^{6x} + c$, where c is a constant.

Problem $\int \log(x)\,dx$

$$= \int \left(1\cdot\log(x) + x\cdot\frac{1}{x} - x\cdot\frac{1}{x}\right)dx$$

$$= \int \left(\frac{d(x\log(x))}{dx} - 1\right)dx$$

Let $F = x$ and $S = \log(x)$. Then $\frac{dF}{dx} = 1$ and $\frac{dS}{dx} = \frac{1}{x}$.

$$= x\log(x) - x + c,$$ where c is a constant.

Problem $\int \frac{1}{2x+7}\,dx$

$$= \int \frac{1}{2}\cdot\frac{1}{2x+7}\cdot 2\,dx$$

Let $J = 2x+7$. Then $\frac{dJ}{dx} = 2$.

$$= \frac{1}{2}\log(2x+7) + c,$$ where c is a constant.

Problem Find $\frac{d}{dx}(\tan(J))$.

$$\frac{d(\tan(J))}{dx} = \frac{d}{dx}\left(\frac{\sin(J)}{\cos(J)}\right) = \frac{d}{dx}\left(\sin(J)\cos(J)^{-1}\right)$$

$$= \sin(J)(-1)\cos(J)^{-2}(-\sin(J))\frac{dJ}{dx} + \cos(J)\frac{dJ}{dx}\cdot\cos(J)^{-1}$$

$$= \frac{+\sin(J)^2}{\cos(J)^2}\frac{dJ}{dx} + \frac{dJ}{dx} = \left(\frac{\sin(J)^2}{\cos(J)^2} + 1\right)\frac{dJ}{dx}$$

$$= \frac{\sin(J)^2 + \cos(J)^2}{\cos(J)^2}\cdot\frac{dJ}{dx} = \frac{1}{\cos(J)^2}\frac{dJ}{dx} = \sec(J)^2\frac{dJ}{dx}$$

Problem
Find $\frac{d}{dx}\arctan(J)$.

Let $y = \arctan(J)$. Then $\tan(y) = J$.

So $\frac{1}{\cos(y)^2}\frac{dy}{dx} = \frac{dJ}{dx}$. So $\frac{dy}{dx} = \cos(y)^2\frac{dJ}{dx}$.

So

$$\frac{dy}{dx} = \frac{\cos(y)^2}{\sin(y)^2+\cos(y)^2}\frac{dJ}{dx} = \frac{1}{\frac{\sin(y)^2}{\cos(y)^2}+\frac{\cos(y)^2}{\cos(y)^2}}\frac{dJ}{dx}$$

$$= \left(\frac{1}{\tan(y)^2+1}\right)\frac{dJ}{dx} = \left(\frac{1}{J^2+1}\right)\frac{dJ}{dx}$$

Problem

$$\int \frac{1}{x^2+2x+2}\,dx$$

$$= \int \frac{1}{x^2+2x+1+1}\,dx = \int \frac{1}{(x+1)^2+1}\,dx$$

Let $J = x+1$. Then $\frac{dJ}{dx} = 1$.

$= \arctan(x+1) + c$, where c is a constant.

Problem

$$\int \frac{x-2}{x^2+2x+2}\,dx$$

$$= \int \frac{(x+1)-3}{(x+1)^2+1}\,dx$$

$$= \int \frac{(x+1)}{(x+1)^2+1}\,dx - 3\int \frac{1}{(x+1)^2+1}\,dx.$$

Let $J_1 = (x+1)^2+1$.
$\frac{dJ_1}{dx} = 2(x+1)$.
Let $J_2 = x+1$.
$\frac{dJ_2}{dx} = 1$.

$$= \int \frac{1}{2}\cdot\frac{1}{(x+1)^2+1}\cdot 2(x+1)\,dx - 3\int \frac{1}{(x+1)^2+1}\,dx$$

$$= \frac{1}{2}\log((x+1)^2+1) - 3\arctan(x+1) + c,$$ where

c is a constant.

Problem $\displaystyle\int \frac{9x+1}{(x-3)(x+1)}\,dx$

$$= \int \frac{9(x+1)-8}{(x-3)(x+1)}\,dx = \int \frac{9(x+1)}{(x-3)(x+1)}\,dx - 8\int \frac{1}{(x-3)(x+1)}\,dx.$$

$$= 9\int \frac{1}{x-3}\,dx - 8\int \frac{1}{-4}\cdot\frac{(x-3)-(x+1)}{(x-3)(x+1)}\,dx.$$

$$= 9\int \frac{1}{x-3}\,dx + 2\left(\int \frac{x-3}{(x-3)(x+1)}\,dx - \int \frac{(x+1)}{(x-3)(x+1)}\,dx\right)$$

$$= 9\int \frac{1}{x-3}\,dx + 2\int \frac{1}{x+1}\,dx - 2\int \frac{1}{x-3}\,dx$$

$$= 7\int \frac{1}{x-3}\,dx + 2\int \frac{1}{x+1}\,dx$$

$$= 7\log(x-3) + 2\log(x+1) + c,$$ where c is

a constant.

Problem $\displaystyle\int \frac{3x^2-2x+1}{(x+1)(x^2+2x+2)}\,dx$

$$= \int \frac{3(x^2+2x+2)-8x-5}{(x+1)(x^2+2x+2)}\,dx$$

$$= \int \frac{3}{x+1}\,dx - \int \frac{8x+5}{(x+1)(x^2+2x+2)}\,dx$$

$$= \int \frac{3}{x+1}\,dx - \int \frac{8(x+1)-3}{(x+1)(x^2+2x+2)}\,dx$$

$$= 3\int \frac{1}{x+1}\,dx - 8\int \frac{1}{x^2+2x+2}\,dx - 3\int \frac{1}{(x+1)(x^2+2x+2)}\,dx$$

$$= 3\int \frac{1}{x+1}\,dx - 8\int \frac{1}{(x+1)^2+1}\,dx - 3\int \frac{(x^2+2x+2)-(x+1)(x+1)}{(x+1)(x^2+2x+2)}\,dx$$

$$= 3\int \frac{1}{x+1}\,dx - 8\int \frac{1}{x^2+2x+2}\,dx - 3\int \left(\frac{1}{x+1} - \frac{x+1}{x^2+2x+2}\right)dx.$$

$$= -8\int \frac{1}{(x+1)^2+1}\,dx + 3\int \frac{1}{2}\cdot\frac{1}{(x+1)^2+1}\cdot 2(x+1)\,dx$$

$$= -8\arctan(x+1) + \frac{3}{2}\log\left((x+1)^2+1\right) + c,$$

where c is a constant.

Problem $\displaystyle\int \frac{1}{x^2+2x+2}\,dx$.

Factor x^2+2x+2. Find roots of x^2+2x+2.

If $x^2+2x+2=0$ then $x=\dfrac{-2\pm\sqrt{2^2-4\cdot 2}}{2} = 1\pm i$.

So $x^2+2x+2 = (x-(1+i))(x-(1-i))$.

So
$$\int \frac{1}{x^2+2x+2}\,dx = \int \frac{1}{(x-(1+i))(x-(1-i))}\,dx$$

$$= \frac{1}{-2i}\int \frac{(x-(1+i))-(x-(1-i))}{(x-(1+i))(x-(1-i))}\,dx$$

$$= \frac{-1}{2i}\left(\int \frac{1}{x-(1-i)}\,dx - \int \frac{1}{x-(1+i)}\,dx\right)$$

$$= \frac{i^2}{2i}\Big(\log(x-(1-i)) + \log(x-(1+i))\Big)+c$$

$$= \tfrac{1}{2}i\log(x-(1-i)) + \tfrac{1}{2}i\log(x-(1+i)) + c,$$

where c is a constant.