27.04.2016 (1)
Calculus Lect. 20
A. Ram

Derivations

J 

$\frac{dJ}{dx}$.

satisfies

(Normalization) $\frac{dx}{dx} = 1$

(Linearity) ~~$\frac{dJ}{dx}$~~ If $c_1, c_2 \in \mathbb{C}$ then

$$\frac{d(c_1 J + c_2 S)}{dx} = c_1 \frac{dJ}{dx} + c_2 \frac{dS}{dx}$$

(Product rule) $\frac{d(JS)}{dx} = J\frac{dS}{dx} + \frac{dJ}{dx} S$.

We proved:

(1) If $c \in \mathbb{C}$ then $\frac{dc}{dx} = 0$

(2) If $n \in \mathbb{Z}$ then $\frac{dJ^n}{dx} = nJ^{n-1}\frac{dJ}{dx}$

(3) $\frac{de^J}{dx} = e^J \frac{dJ}{dx}$ $\qquad \frac{d\log(J)}{dx} = \frac{1}{J}\frac{dJ}{dx}$.

$\frac{d\sin(J)}{dx} = \cos(J)\frac{dJ}{dx}$ $\qquad \frac{d\arcsin(J)}{dx} = \left(\frac{1}{\sqrt{1-J^2}}\right)\frac{dJ}{dx}$

$\frac{d\cos(J)}{dx} = -\sin(J)\frac{dJ}{dx}$

$\frac{d\tan(J)}{dx} = \frac{1}{\cos(J)^2}\frac{dJ}{dx}$. $\qquad \frac{d\arctan(J)}{dx} = \left(\frac{1}{J^2+1}\right)\frac{dJ}{dx}$

<u>Problem</u> Let $y = \arctan(J)$. Find $\frac{dy}{dx}$.

Then $\tan(y) = J$.

So $\frac{d\tan(y)}{dx} = \frac{dJ}{dx}$ and $\frac{1}{\cos(y)^2}\frac{dy}{dx} = \frac{dJ}{dx}$.

So $$\frac{dy}{dx} = \cos(y)^2\frac{dJ}{dx} = \left(\frac{\cos(y)^2}{\sin(y)^2+\cos(y)^2}\right)\frac{dJ}{dx}$$

$$= \left(\frac{1}{\frac{\sin(y)^2}{\cos(y)^2}+\frac{\cos(y)^2}{\cos(y)^2}}\right)\frac{dJ}{dx} = \left(\frac{1}{\tan(y)^2+1}\right)\frac{dJ}{dx}$$

$$= \left(\frac{1}{J^2+1}\right)\frac{dJ}{dx}$$

<u>Problem</u> Let $y = \arcsin(J)$. Find $\frac{dy}{dx}$.

Since $\sin(y) = J$ then $\frac{d\sin(y)}{dx} = \frac{dJ}{dx}$.

So $\cos(y)\frac{dy}{dx} = \frac{dJ}{dx}$ and $\frac{dy}{dx} = \frac{1}{\cos(y)}\frac{dJ}{dx}$

So
$$\frac{dy}{dx} = \frac{1}{\sqrt{\cos(y)^2}}\frac{dJ}{dx} = \frac{1}{\sqrt{1-\sin(y)^2}}\frac{dJ}{dx}$$

$$= \left(\frac{1}{\sqrt{1-J^2}}\right)\frac{dJ}{dx}$$

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Calculus Lect. 2 (3)
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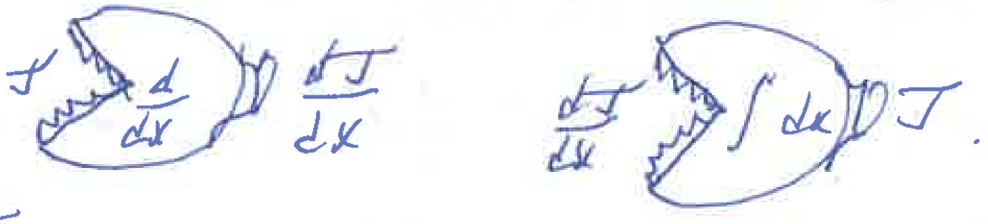
Backwards

Backwards of addition is subtraction.

Backwards of multiplication is division

Backwards of differentiation is integration



Since

$$\int 0\,dx = 0 \text{ and } \int 0\,dx = 1 \text{ and } \int 0\,dx = 6257.$$

then

$$\int 0\,dx = c, \text{ where } c \text{ is a constant}$$

and

$$\int \frac{dJ}{dx}\,dx = J + c, \text{ where } c \text{ is a constant.}$$

Problem $\int \sin(x)^4 \cos(x)\,dx$

Let $J = \sin(x)$.

Then $\frac{dJ}{dx} = \cos(x)$.

$$= \int J^4 \frac{dJ}{dx}\,dx$$

$$= \int \frac{1}{5}\, 5J^4 \frac{dJ}{dx}\,dx = \frac{1}{5}\int \frac{dJ^5}{dx}\,dx$$

$$= \frac{1}{5}J^5 + c = \frac{1}{5}\sin(x)^5 + c,$$

where c is a constant.

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<u>Problem</u>

$$\int \frac{\sin(\log(x))}{x}\,dx$$

Let $J = \log(x)$

Then $\frac{dJ}{dx} = \frac{1}{x}$.

$$= \int \sin(\log(x))\cdot\frac{1}{x}\,dx$$

$$= \int \sin(J)\frac{dJ}{dx}\,dx = \int -\left(-\sin(J)\frac{dJ}{dx}\right)dx$$

$$= -\int \frac{d\cos(J)}{dx}\,dx = -\cos(J)+c$$

$$= -\cos(\log(x))+c,$$ where c is a constant

Check: $\frac{d(-\cos(\log(x)))}{dx} = -(-\sin(\log(x)))\frac{d(\log(x))}{dx} = \sin(\log(x))\frac{1}{x}$.

<u>Problem</u>

$$\int 2xe^{-x^2}\,dx$$

Let $J = -x^2$

Then $\frac{dJ}{dx} = -2x$.

$$= \int -e^{-x^2}(-2x)\,dx$$

$$= \int -e^{J}\frac{dJ}{dx}\,dx$$

$$= -\int \frac{de^{J}}{dx}\,dx = -e^{J}+c = -e^{-x^2}+c,$$

where c is a constant.

Check: $\frac{d(-e^{-x^2}+c)}{dx} = -\frac{de^{-x^2}}{dx} + \frac{dc}{dx} = -e^{-x^2}(-2x)+0$

$$= 2xe^{-x^2}.$$

Problem $\int (2x+1)\sqrt{x-3}\,dx$

$$= \int (2(x-3)+7)(x-3)^{\frac{1}{2}}dx$$

$$= \int \left(2(x-3)^{3/2} + 7(x-3)^{\frac{1}{2}}\right) dx$$

$$= 2\cdot\frac{2}{5}(x-3)^{5/2} + 7\cdot\frac{2}{3}(x-3)^{3/2} + c,$$ where c is a constant.

Check: $$\frac{d\left(\frac{4}{5}(x-3)^{5/2}+\frac{14}{3}(x-3)^{3/2}+c\right)}{dx}$$

$$= \frac{4}{5}\cdot\frac{5}{2}(x-3)^{3/2}\cdot 1 + \frac{14}{3}\cdot\frac{3}{2}(x-3)^{\frac{1}{2}}\cdot 1 + 0$$

$$= 2(x-3)^{3/2} + 7(x-3)^{\frac{1}{2}} = (2(x-3)+7)(x-3)^{\frac{1}{2}}$$

$$= (2x+1)(x-3)^{\frac{1}{2}}.$$

Problem $\int \sin(x)^{25}\cos(x)^5\,dx$

Let $J = \sin(x)$

Then $\frac{dJ}{dx} = \cos(x)$

$$= \int \sin(x)^{25}\left(\cos(x)^2\right)^2\cos(x)\,dx$$

$$= \int \sin(x)^{25}\left(1-\sin(x)^2\right)^2\cos(x)\,dx$$

$$= \int \sin(x)^{25}\left(1-2\sin(x)^2+\sin(x)^4\right)\cos(x)\,dx$$

$$= \int \left(\sin(x)^{24}\cos(x) - 2\sin(x)^{26}\cos(x) + \sin(x)^{29}\cos(x)\right)dx$$

$$= \int \left(J^{24} - 2J^{26} + J^{29}\right)\frac{dJ}{dx}\,dx$$

$$= \frac{1}{25}J^{25} - \frac{2}{27}J^{27} + \frac{1}{30}J^{30} + c,$$ where c is a constant.