<u>Problem</u> Prove that $\frac{d\sin(J)}{dx} = \cos(J)\frac{dJ}{dx}$.

$$\frac{d\sin(J)}{dx} = \frac{d\,\frac{1}{2i}(e^{iJ}-e^{-iJ})}{dx} = \frac{1}{2i}\,\frac{d(e^{iJ}-e^{-iJ})}{dx}$$

$$= \frac{1}{2i}\left(\frac{de^{iJ}}{dx} - \frac{d(e^{-iJ})}{dx}\right) = \frac{1}{2i}\left(e^{iJ}\frac{d(iJ)}{dx} - e^{-iJ}\frac{d(-iJ)}{dx}\right)$$

$$= \frac{1}{2i}\left(e^{iJ}\cdot i\cdot\frac{dJ}{dx} - e^{-iJ}\cdot(-i)\cdot\frac{dJ}{dx}\right)$$

$$= \frac{i}{2i}(e^{iJ}+e^{-iJ})\frac{dJ}{dx} = \frac{1}{2}(e^{iJ}+e^{-iJ})\frac{dJ}{dx}$$

$$= \cos(J)\frac{dJ}{dx}.$$

<u>Problem 3.10 (h)</u> Let $4\cos(x)\sin(y) = 1$. Find $\frac{dy}{dx}$.

Since $\cos(x)\sin(y) = \frac{1}{4}$ then

$$\frac{d(\cos(x)\sin(y))}{dx} = \frac{d(\frac{1}{4})}{dx} = 0.$$

So

$$0 = \cos(x)\frac{d\sin(y)}{dx} + \frac{d\cos(x)}{dx}\sin(y)$$

$$= \cos(x)\cos(y)\frac{dy}{dx} + (-\sin(x))\frac{dx}{dx}\sin(y)$$

$$= \cos(x)\cos(y)\frac{dy}{dx} - \sin(x)\sin(y).$$

02.04.2026
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So $\cos(x)\cos(y)\frac{dy}{dx} = \sin(x)\sin(y)$.

So $\frac{dy}{dx} = \frac{\sin(x)\sin(y)}{\cos(x)\cos(y)} = \tan(x)\tan(y)$.

Problem Prove that $\frac{d\log(J)}{dx} = \frac{1}{J}\frac{dJ}{dx}$.

Let $y = \log(J)$. Then $e^y = J$.

So $\frac{de^y}{dx} = \frac{dJ}{dx}$. So $e^y\frac{dy}{dx} = \frac{dJ}{dx}$.

So $\frac{dy}{dx} = \frac{1}{e^y}\frac{dJ}{dx} = \frac{1}{J}\frac{dJ}{dx}$.

Problem Find $\frac{d\cot(J)}{dx}$.

$$\frac{d\cot(J)}{dx} = \frac{d\left(\frac{\cos(J)}{\sin(J)}\right)}{dx} = \frac{d\cos(J)\sin(J)^{-1}}{dx}.$$

$$= \cos(J)\frac{d\sin(J)^{-1}}{dx} + \frac{d\cos(J)}{dx}\sin(J)^{-1}.$$

$$= \cos(J)\cdot(-1)\sin(J)^{-2}\frac{d\sin(J)}{dx} + (-\sin(J))\frac{dJ}{dx}\sin(J)^{-1}.$$

$$= \left(-\frac{\cos(J)^2}{\sin(J)^2} - \frac{\sin(J)}{\sin(J)}\right)\frac{dJ}{dx}$$

$$= \left(-\frac{\cos(J)^2 - \sin(J)^2}{\sin(J)^2}\right)\frac{dJ}{dx}$$

$$= \frac{-1}{\sin(J)^2}\left(\cos(J)^2 + \sin(J)^2\right)\frac{dJ}{dx} = -\csc(J)^2\frac{dJ}{dx}.$$

Problem 3.10 (4) Let $x^2 y = e^{xy}$. Find $\frac{dy}{dx}$.

Since $\frac{d(x^2 y)}{dx} = \frac{d(e^{xy})}{dx}$ then $x^2 \frac{dy}{dx} + \frac{dx^2}{dx} y = e^{xy} \frac{d(xy)}{dx}$.

So $x^2 \frac{dy}{dx} + 2x \frac{dx}{dx} y = e^{xy}\left(x \frac{dy}{dx} + \frac{dx}{dx} y\right)$

So $x^2 \frac{dy}{dx} + 2xy = e^{xy}\left(x \frac{dy}{dx} + y\right)$

So $x^2 \frac{dy}{dx} + 2xy = e^{xy} x \frac{dy}{dx} + e^{xy} \cdot y$

So $(x^2 - xe^{xy}) \frac{dy}{dx} = ye^{xy} - 2xy$

So $\frac{dy}{dx} = \frac{ye^{xy} - 2xy}{x^2 - xe^{xy}}$

Problem Find $\frac{d\sin(J)}{dx}$.

$\frac{d\sin(J)}{dx} = \frac{d}{dx} \frac{1}{2i}(e^{iJ} - e^{-iJ}) = \frac{1}{2i}\left(\frac{de^{iJ}}{dx} - \frac{de^{-iJ}}{dx}\right)$

$= \frac{1}{2i}\left(e^{iJ} \frac{d(iJ)}{dx} - e^{-iJ} \frac{d(-iJ)}{dx}\right)$