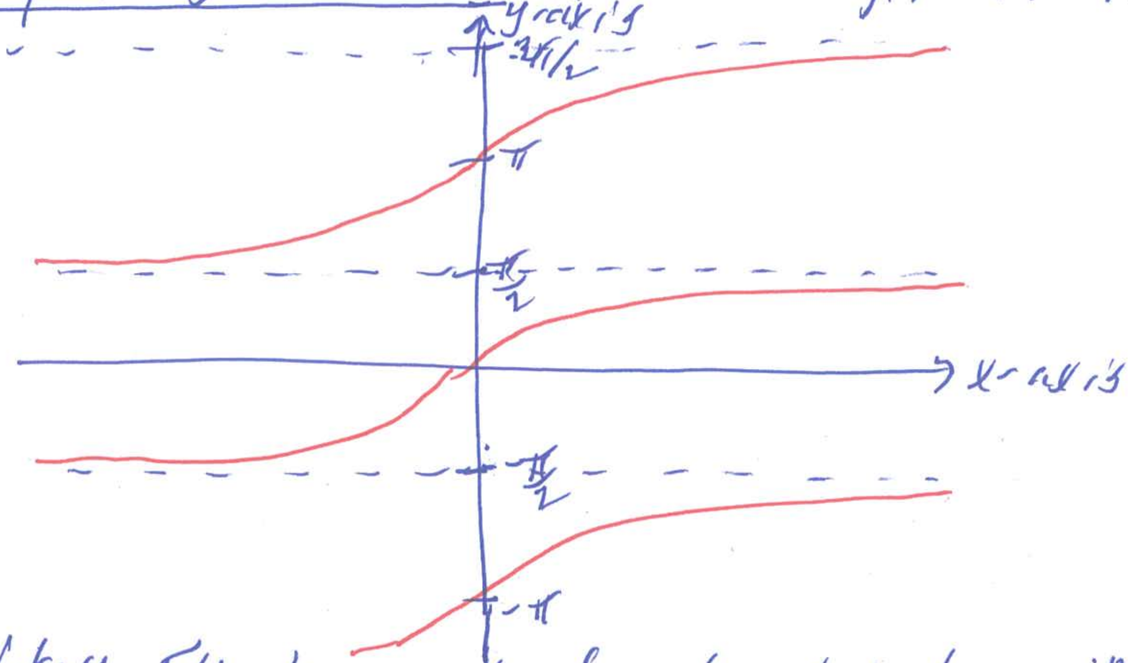
18.06.2026 (3)

Graph of $y = \arctan(x)$ (or $x = \tan(y)$)

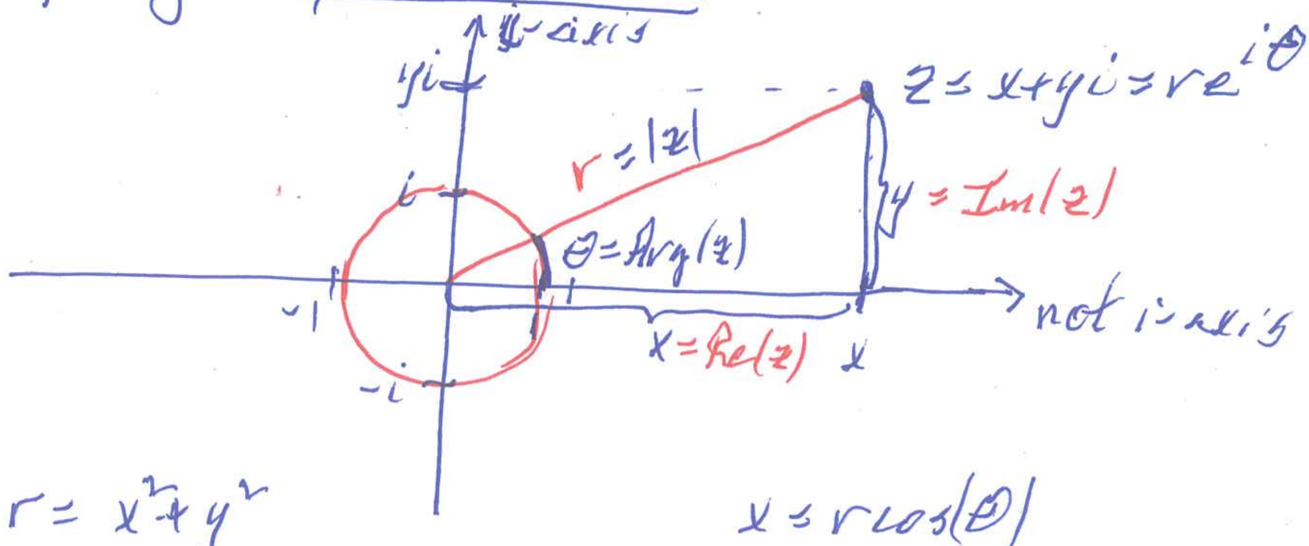
Calculus Lect. 7

A. Ram



Notes: Flip the graph of $y = \tan(x)$ to switch the x and y axis.

Graphing complex numbers



$r = x^2 + y^2$

$\theta = \arctan\left(\frac{y}{x}\right)$

$x = r\cos(\theta)$

$y = r\sin(\theta)$

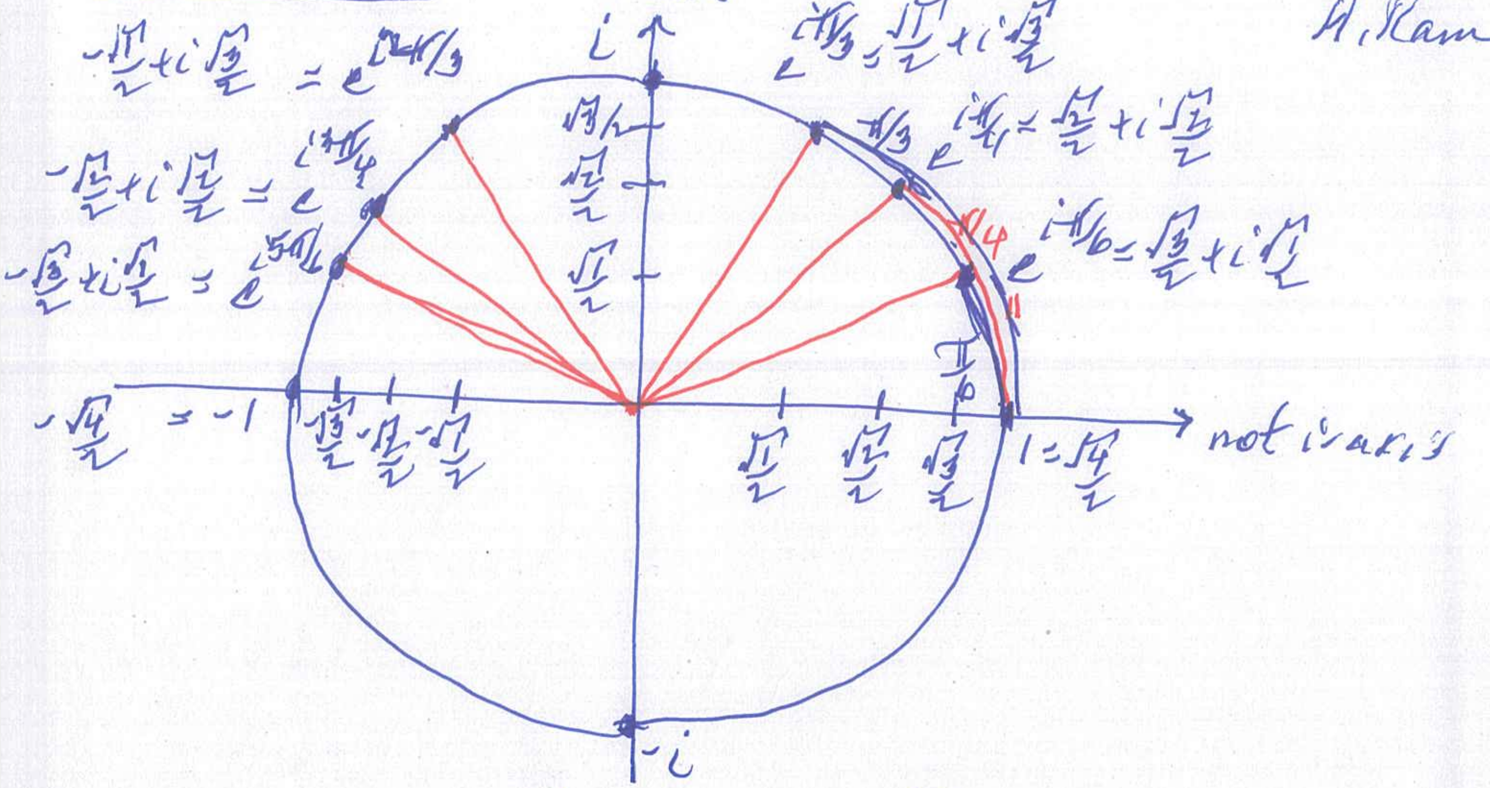
$$\boxed{re^{i\theta} = r\cos(\theta) + ir\sin(\theta)}$$

Euler's formula

So the unit circle is

$$\left\{ x+iy \;\middle|\; \begin{matrix} x, y \in \mathbb{R} \text{ and} \\ x^2+y^2=1 \end{matrix} \right\} = \left\{ e^{i\theta} \mid \theta \in \mathbb{R}_{[-\pi,\pi]} \right\}$$

The Tattoo



Let $z_1 = r_1 e^{i\theta_1}$ and $z_2 = e^{i\theta_2}$.

Since $r_1 e^{i\theta_1} \cdot r_2 e^{i\theta_2} = r_1 r_2 e^{i(\theta_1+\theta_2)}$ then

$|z_1 z_2| = |z_1|\cdot|z_2|$ and $Arg(z_1 z_2) = Arg(z_1) + Arg(z_2)$

The conjugate of $z = x+iy = re^{i\theta}$ is

$\bar{z} = x - iy = re^{-i\theta}$

Then

$z\bar{z} = (x+iy)(x-iy) = x^2+y^2 = |z|^2$

$= re^{i\theta}\, re^{-i\theta} = r^2 e^{i0} = r^2$

19.03.2026

Calculus Lect 1 8

A. Ram

Problem 1.5

$$\frac{(3+4i)(1-i)}{2+2i} = \frac{3-3i+4i-4i^2}{2+2i} = \frac{3+i+4}{2+2i}$$

$$= \frac{(7+i)}{(2+2i)}\frac{(2-2i)}{(2-2i)} = \frac{14-14i+2i-2i^2}{4-4i+4i-4i^2}$$

$$= \frac{14-12i+2}{4+4} = \frac{16-12i}{8} = 2-\frac{3}{2}i.$$

So $Re\left(\frac{(3+4i)(1-i)}{(2+2i)}\right) = 2$, $Im\left(\frac{(3+4i)(1-i)}{(2+2i)}\right) = -\frac{3}{2}$

$$\left|\frac{(3+4i)(1-i)}{(2+2i)}\right| = \frac{|3+4i|\cdot|1-i|}{|2+2i|} = \frac{\sqrt{3^2+4^2}\cdot\sqrt{1^2+(-1)^2}}{\sqrt{2^2+2^2}}$$

$$= \frac{\sqrt{9+16}\cdot\sqrt{1+1}}{\sqrt{4+4}} = \frac{\sqrt{25}\sqrt{2}}{\sqrt{4}\sqrt{2}} = \frac{5}{2}.$$

Check: $$\left|\frac{(3+4i)(1-i)}{(2+2i)}\right| = \left|2-\frac{3}{2}i\right| = \sqrt{2^2+\left(\frac{3}{2}\right)^2} = \sqrt{4+\frac{9}{4}}$$

$$= \sqrt{\frac{16+9}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{2}.$$

19.03.2026 ②

Calculus Lect 8

A. Ram

Problem 2.11 $z = \sqrt{3} + i$ is

$$z = 2\cdot\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = 2\cdot\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{1}}{2}i\right) = 2e^{i\pi/6}.$$

So $|z| = 2$ and $\mathrm{Arg}(z) = \frac{\pi}{6}$.

Check: $|z| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$.

$$\mathrm{Arg}(z) = \arctan\left(\frac{1}{\sqrt{3}}\right) = \arctan\left(\frac{1/2}{\sqrt{3}/2}\right)$$

$$= \arctan\left(\frac{\sin\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)}\right) = \arctan\left(\tan\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{6}.$$

The most important expression in math:

$$e^x = 1 + x + \frac{1}{2\cdot 1}x^2 + \frac{1}{3\cdot 2\cdot 1}x^3 + \frac{1}{4\cdot 3\cdot 2\cdot 1}x^4 + \dots$$

$$= 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots$$

The most important theorem in math!

$$e^x e^y = e^{x+y}$$