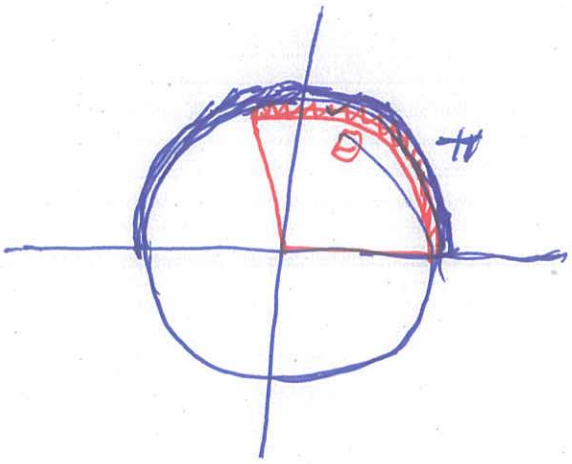
What is π? What is an angle?

π is the distance halfway around a circle of radius 1

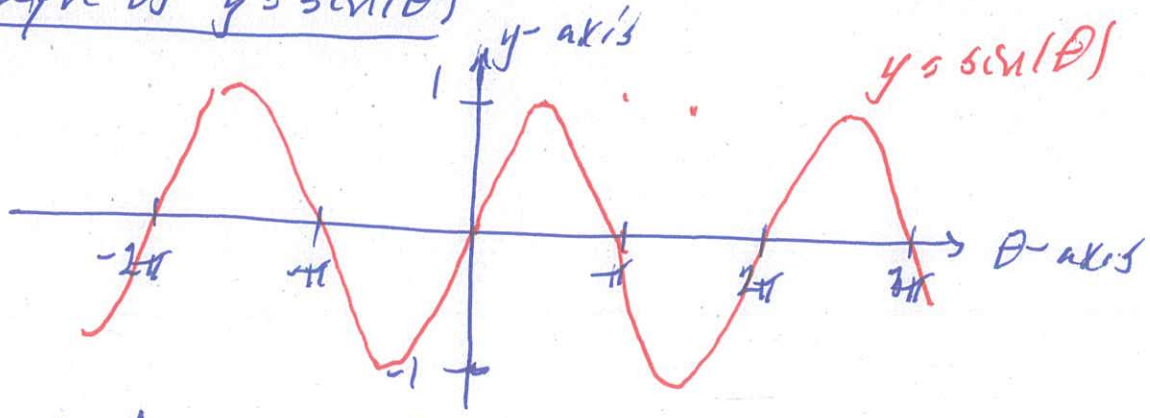
Measure angles by distance traveled on a circle of radius 1



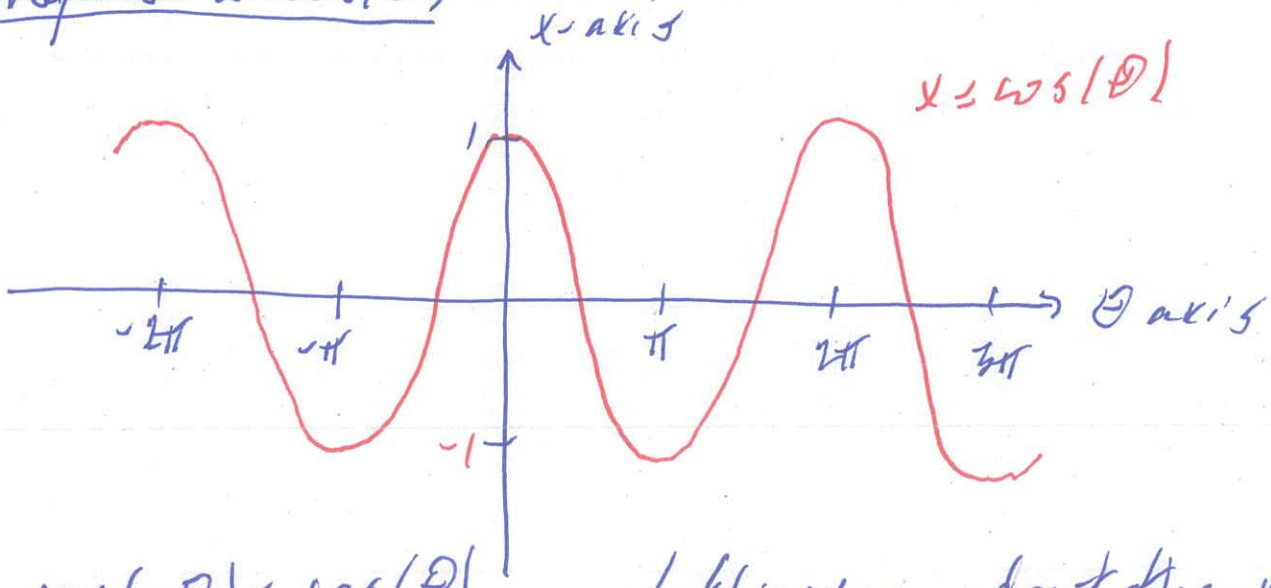
$\cos(\theta)$ = x-coordinate at angle θ from $(1,0)$

$\sin(\theta)$ = y-coordinate at angle θ from $(1,0)$.

Graph of $y = \sin(\theta)$

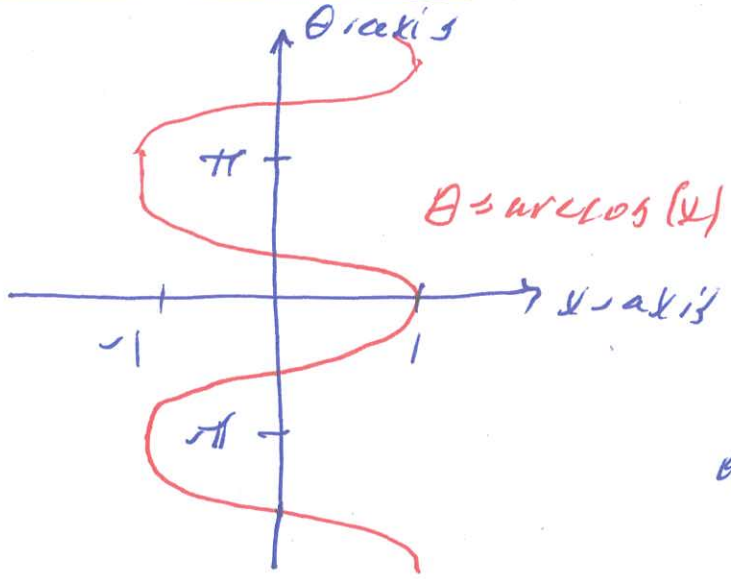


Graph of $x = \cos(\theta)$



$\cos(-\theta) = \cos(\theta)$ (flipping about the x-axis)

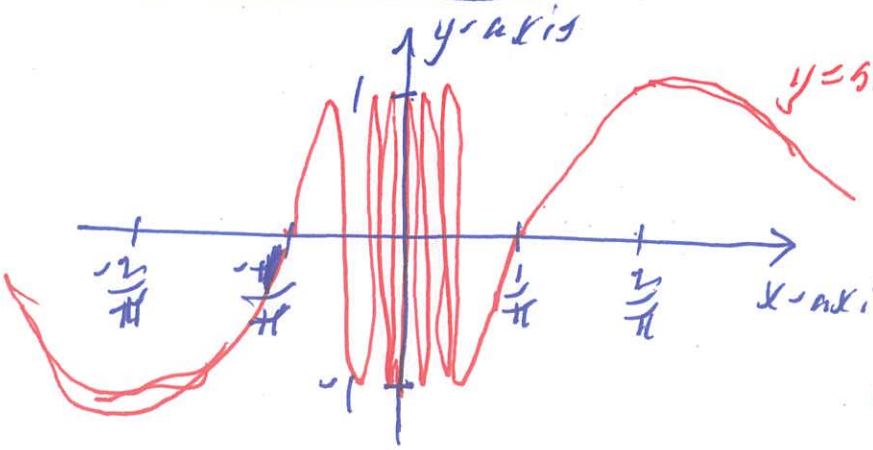
Graph of $\theta = \arccos(|x|)$



Notes:

Flip the graph of $x = \cos(|\theta|)$ to switch positions of the x-axis and θ-axis

Graph of $y = \sin\left(\frac{1}{x}\right)$

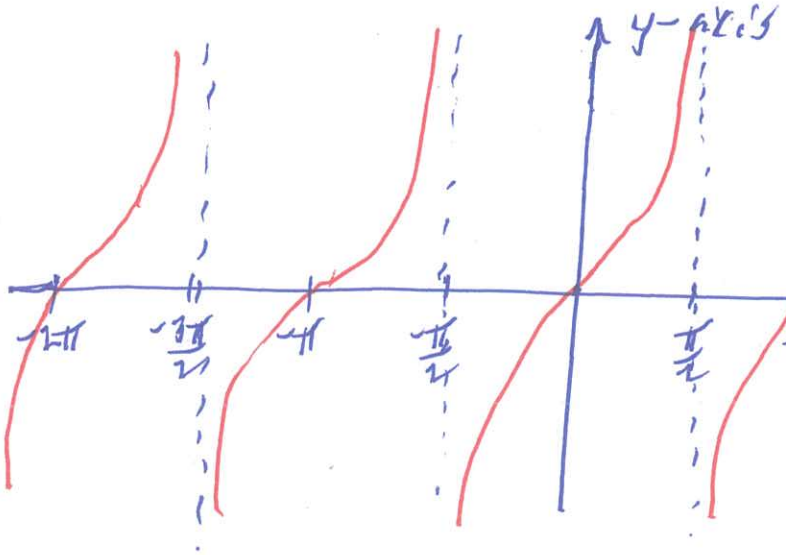


Notes:

(a) Flip the positive half of the graph of $y = \sin(|x|)$ about $x = 1$

(b) Flip the negative half of the graph of $y = \sin(|x|)$ about $x = -1$.

Graph of $y = \tan(x) = \frac{\sin(x)}{\cos(x)}$



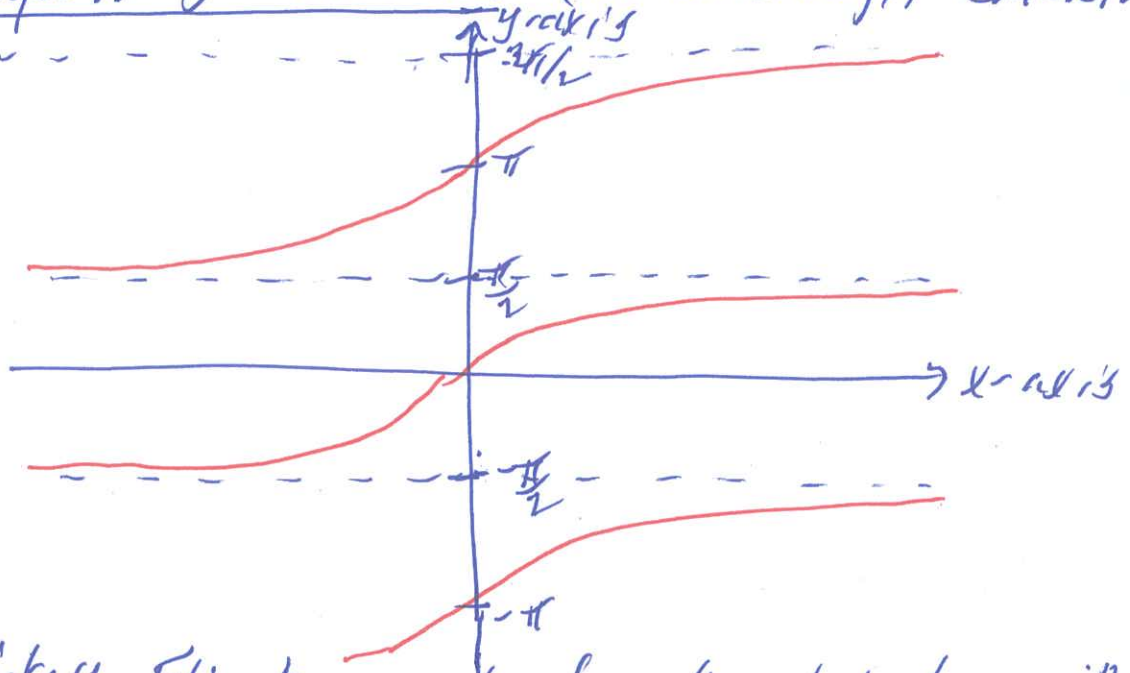
Notes:

(a) $\tan(0) = 0$
$\tan(\pi) = 0$
$\tan(\frac{\pi}{2})$ undefined in $\mathbb{R}$.

(b) Periodic repetition every π.

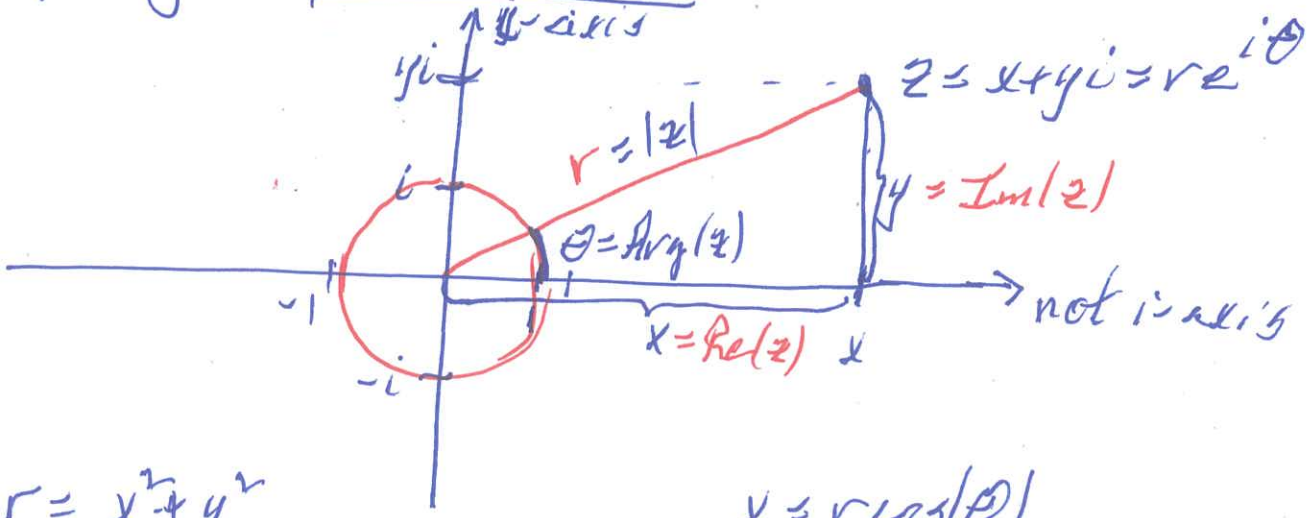
Graph of $y = \arctan(x)$ (or $x \tan y$) Calculus Lect. 7

D. Ram



Notes: Flip the graph of $y = \tan(x)$ to switch the x and y axes.

Graphing complex numbers



$r = x^2 + y^2$ $\qquad$ $x = r\cos(\theta)$

$\theta = \arctan\left(\frac{y}{x}\right)$ $\qquad$ $y = r\sin(\theta)$

$$\boxed{re^{i\theta} = r\cos(\theta) + ir\sin(\theta)}$$ Euler's formula

So the unit circle is

$$\{x + iy \mid x, y \in \mathbb{R} \text{ and } x^2 + y^2 = 1\} = \{e^{i\theta} \mid \theta \in \mathbb{R}_{(-\pi, \pi]}\}$$