# Graphs of inverse functions (Flipping)

Calculus sheet 6
A. Ram

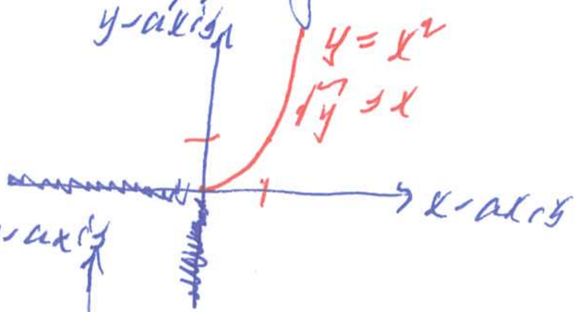
## sq and sqrt

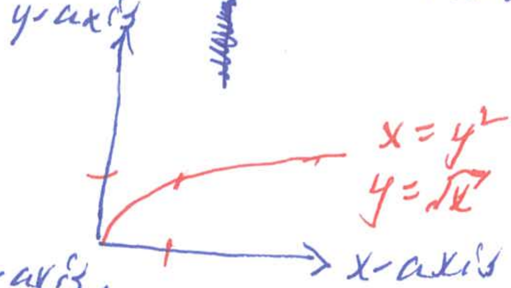
$sq: \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$

$x \mapsto x^2$

$sqrt: \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$

$x \mapsto \sqrt{x}$



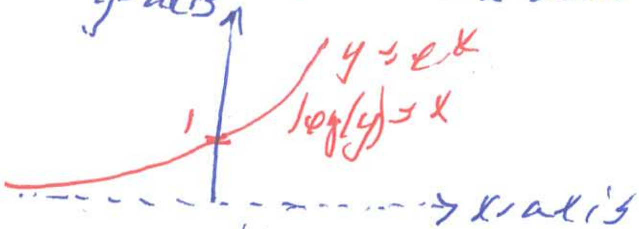

## exp and log

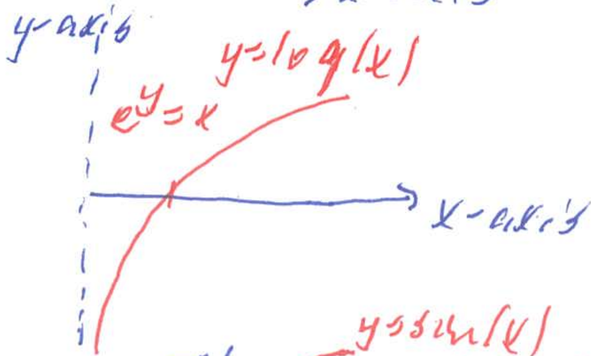
$exp: \mathbb{R} \to \mathbb{R}_{>0}$

$x \mapsto e^x$

$log: \mathbb{R}_{>0} \to \mathbb{R}$

$x \mapsto \log(x)$



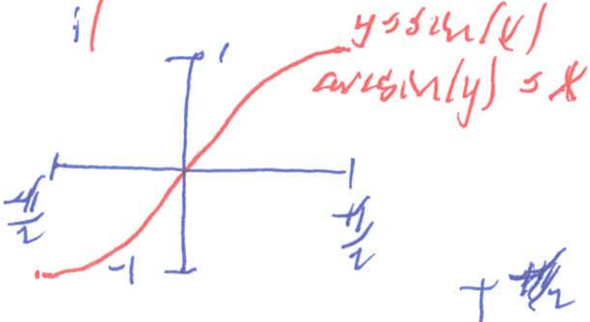

## sin and arcsin

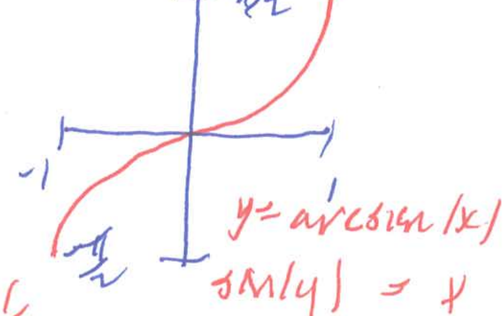
$\sin: \mathbb{R}_{[-\frac{\pi}{2}, \frac{\pi}{2}]} \to \mathbb{R}_{[-1,1]}$

$x \mapsto \sin(x)$

$\arcsin: \mathbb{R}_{[-1,1]} \to \mathbb{R}_{[-\frac{\pi}{2}, \frac{\pi}{2}]}$

$x \mapsto \arcsin(x)$



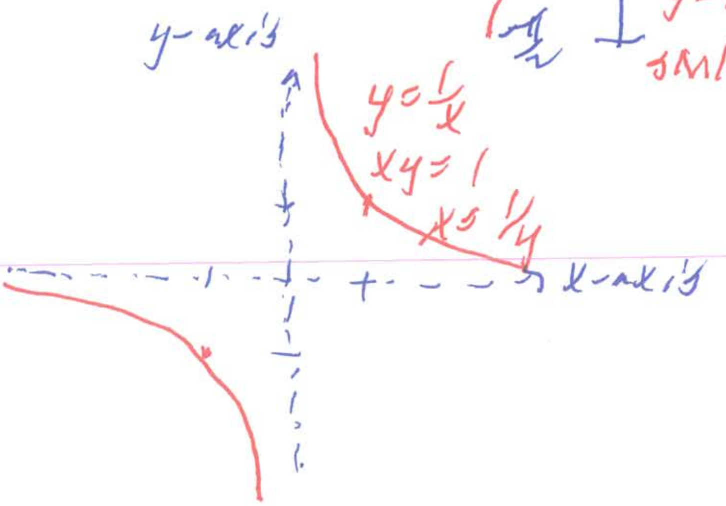

## $y = \frac{1}{x}$ and $x = \frac{1}{y}$

$\mathbb{R}_{\neq 0} \to \mathbb{R}_{\neq 0}$

$x \mapsto \frac{1}{x}$

$\mathbb{R}_{\neq 0} \to \mathbb{R}_{\neq 0}$

$x \mapsto \frac{1}{x}$



Problem 1.38 Find the implied domain of the following expressions.

(1) $\varphi = \sqrt{\dfrac{x-3}{x+1}}$ (3) $\varphi = \log(\sqrt{2x+5}-1)$

(5) $\varphi = \sqrt{\dfrac{1}{x} - \dfrac{1}{x-2}}$

Let $\varphi$ be an expression. The implied domain of $\varphi$ is the largest subset $S$ of $\mathbb{R}$ such that there exists $T \subseteq \mathbb{R}$ such that

$$\begin{array}{rcl} S & \to & T \\ x & \mapsto & \varphi(x) \end{array}$$ is a function.

(1) $\begin{array}{rcl} \mathbb{R}_{\geq 0} & \to & \mathbb{R}_{\geq 0} \\ x & \mapsto & \sqrt{x} \end{array}$ is a function

$\begin{array}{rcl} \mathbb{R}_{\neq -1} & \to & \mathbb{R} \\ x & \mapsto & \dfrac{x-3}{x+1} \end{array}$ is a function.

For $\varphi = \sqrt{\dfrac{x-3}{x-1}}$ let $S = \{x \in \mathbb{R} \mid x \neq -1 \text{ and } \dfrac{x-3}{x+1} \geq 0\}$.

Then $\begin{array}{rcl} S & \to & \mathbb{R}_{\geq 0} \\ x & \mapsto & \sqrt{\dfrac{x-3}{x+1}} \end{array}$ is a function.

Note: $S = \{x \in \mathbb{R} \mid x \neq -1 \text{ and } \dfrac{x-3}{x+1} \geq 0\}$

$= \{x \in \mathbb{R} \mid x \neq -1 \text{ and } x-3 \geq 0 \cdot (x+1) \text{ and } x+1 > 0\}$

$\cup \{x \in \mathbb{R} \mid x \neq -1 \text{ and } x-3 \leq 0 \cdot (x+1) \text{ and } x+1 < 0\}$

So $S = \{x\in\mathbb{R} \mid x \geq 3 \text{ and } x > -1\}$

$\quad \cup \{x\in\mathbb{R} \mid x \leq 3 \text{ and } x < -1\}$

$\quad = \mathbb{R}_{[3,\infty)} \cup \mathbb{R}_{(-\infty,-1)} = [3,\infty) \cup (-\infty,-1).$

(3) $\varphi = \log(\sqrt{2x+5}-1)$

$$\begin{array}{rcl} \mathbb{R}_{>0} & \to & \mathbb{R} \\ x & \mapsto & \log(x) \end{array} \quad \text{is a function}$$

Let $S = \{x\in\mathbb{R} \mid 2x+5 \geq 0 \text{ and } \sqrt{2x+5}-1 > 0\}$

then

$$\begin{array}{rcl} S & \to & \mathbb{R} \\ x & \mapsto & \log(\sqrt{2x+5}-1) \end{array} \quad \text{is a function}$$

<u>Note</u>! $S = \{x\in\mathbb{R} \mid 2x \geq -5 \text{ and } \sqrt{2x+5} > 1\}$

$\quad = \{x\in\mathbb{R} \mid 2x \geq -5 \text{ and } 2x+5 > 1^2\}$

$\quad = \{x\in\mathbb{R} \mid 2x \geq -5 \text{ and } 2x > -4\}$

$\quad = \{x\in\mathbb{R} \mid 2x > -4\} = \{x\in\mathbb{R} \mid x > -2\} = (-2,\infty).$

(5) $\varphi = \sqrt{\frac{1}{x} - \frac{1}{x-2}}$

$$\begin{array}{rcl} \mathbb{R}_{\neq 0} & \to & \mathbb{R}_{\neq 0} \\ x & \mapsto & \frac{1}{x} \end{array} \quad \text{is a function}$$

$$\begin{array}{rcl} \mathbb{R}_{\neq 2} & \to & \mathbb{R} \\ x & \mapsto & \frac{1}{x-2} \end{array} \quad \text{is a function}$$

Let $S = \{x\in\mathbb{R} \mid x \neq 0,\ x \neq 2 \text{ and } \frac{1}{x} - \frac{1}{x-2} \geq 0\}$

Then

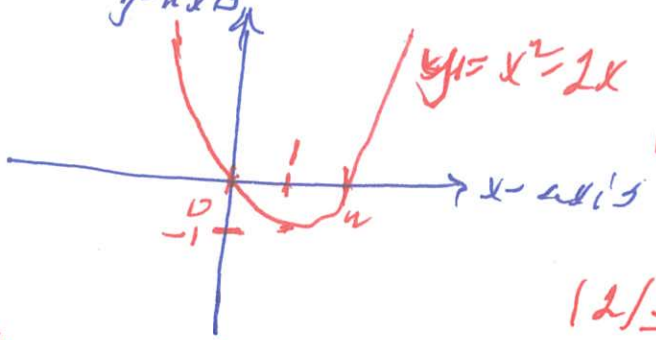
$$\begin{array}{rcl} S & \to & \mathbb{R}_{\geq 0} \\ x & \mapsto & \sqrt{\frac{1}{x} - \frac{1}{x-2}} \end{array} \quad \text{is a function}$$

Note:

$S = \{x \in \mathbb{R} \mid x \neq 0,\ x \neq 2 \text{ and } \frac{1}{x} - \frac{1}{x-2} \geq 0\}$

$= \{x \in \mathbb{R} \mid x \neq 0, x \neq 2 \text{ and } \frac{x-2-x}{x(x-2)} \geq 0\}$

$= \{x \in \mathbb{R} \mid x \neq 0, x \neq 2 \text{ and } \frac{-2}{x^2-2x} \geq 0\}$




Notes:

(1) $y = x^2 - 2x$ is a concave up parabola.

(2) If $x = 0$ then $y = 0^2 - 2\cdot 0 = 0$

(3) If $x = 2$ then $y = 2^2 - 2\cdot 2 = 0$

(4) If $x = 1$ then $y = 1^2 - 2\cdot 1 = -1$.

Then

$S = \{x \in \mathbb{R} \mid x \neq 0, x \neq 2 \text{ and } -2 \geq 0 \text{ and } x^2 - 2x > 0\}$

$\cup \{x \in \mathbb{R} \mid x \neq 0, x \neq 2 \text{ and } -2 \leq 0 \text{ and } x^2 - 2x < 0\}$

$= \emptyset \cup \mathbb{R}_{(0,2)} = (0, 2).$