05.03.2026 ①
Calculus Lect. 3
A. Ram

Three types of proofs

(I) LHS = RHS

Proof:

LHS = ... = ... = ... = stuff.

RHS = ... = ... = ... = THE SAME stuff

So LHS = RHS

(II) To show: If ___A___ then ___B___

Proof: Assume ___A___

To show: ___B___

⋮

(III) there exists ___C___ such that ___D___

Proof Let C = ______

To show: C satisfies ___D___.

⋮

Negations

Statement: If ___A___ then ___B___

Negation: If ___A___ then ___not B___

Converses

Statement: If ___A___ then ___B___

Converse: If ___B___ then ___A___

Contrapositives:

Statement: If $\underline{A}$ then $\underline{B}$

Contrapositive: If $\underline{\text{not } B}$ then $\underline{\text{not } A}$

Problem 1.7: Negation

(1) If $x \in \mathbb{R}$ then $\sin(x) \le 1$.

Negation: If $x \in \mathbb{R}$ then $\sin(x) \not\le 1$.

(2) There exists $n \in \mathbb{N}$ such that $n^2 > n$

Negation: There does not exist $n \in \mathbb{N}$ such that $n^2 > n$

(3) If $n \in \mathbb{Z}$ and n is divisible by 4 then n is even.

Negation: If $n \in \mathbb{Z}$ and n is divisible by 4 then n is not even.

Problem 1.18 and Exercise 1.8 Converses.

(1) If n is divisible by 4 then n is even

Converse: If n is even then n is divisible by 4.

(2) If $A \subseteq B$ then $A \cup B = B$

Converse: If $A \cup B = B$ then $A \subseteq B$.

(3) If f is monotone increasing then f is injective

Converse: If f is injective then f is monotone increasing

(4) If $x > 3$ then $x^2 > 9$

Converse: If $x^2 > 9$ then $x > 3$.

(5) If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

Converse: If $A \subseteq C$ then $A \subseteq B$ and $B \subseteq C$.

Problem 1.23 Contrapositives

(1) Assume $x \in \mathbb{R}$. If $x > 2$ then $x^2 > 4$

Contrapositive: Assume $x \in \mathbb{R}$. If $x^2 \not> 4$ then $x \not> 2$

(2) Assume $n \in \mathbb{Z}$. If n is divisible by 6 then n is even

Contrapositive: Assume $n \in \mathbb{Z}$. If n is odd then n is not divisible by 6.

The contrapositive is the same logical statement just said in a different way.

05.03.2026 Calculus Lect. 3 A. Ram (4)

Problem 1.4 Notation $\forall, \exists, \neg$

Translation: Math to English

Let $p(x) =$ "x is divisible by 1"
$q(x) =$ "x is divisible by 2".

(1) $\forall x \in \mathbb{N}\ p(x)$

English: If $x \in \mathbb{Z}_{>0}$ then x is divisible by 1.

(2) $\exists x \in \mathbb{N}\ p(x)$

English: There exist $x \in \mathbb{Z}_{>0}$ such that x is divisible by 1.

(3) $\forall x \in \mathbb{N}\ q(x)$

English: If $x \in \mathbb{Z}_{>0}$ then x is divisible by 2.

(4) $\forall x \in \mathbb{N}\ (q(x) \wedge p(x))$

English: If $x \in \mathbb{Z}_{>0}$ then x is divisible by 1 and x is divisible by 2.

(5) $\exists x \in \mathbb{N} \neg p(x)$

English: there exists $x \in \mathbb{Z}_{>0}$ such that x is not divisible by 1.