Calculus Lect. 1
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Problems on Sets

Problem 1.14 $A=\{1,2,3\}$ and $B=\{2,3,4\}$

$A\cap B=\{2,3\}$

$A\cup B=\{1,2,3,4\}$

A is not a subset of B since $1\in A$ and $1\notin B$

A set is a collection of elements.

$S\cap T=\{u \mid u\in S \text{ and } u\in T\}$

$S\cup T=\{u \mid u\in S \text{ or } u\in T\}$

A *subset* of S is a set A such that if $a\in A$ then $a\in S$. Write $A\subseteq S$.

Problem 1.15 $U=\{1,2,3,4,5\}$ and $A=\{2,4\}$

$U\setminus A=\{1,3,5\}$

$\emptyset\subseteq A$

$A\not\subseteq\emptyset$ since $2\in A$ and $2\notin\emptyset$

$U\setminus A=\{x\in U \mid x\notin A\}$

The *emptyset* $\emptyset$ is the set with no elements

Problem 1.18 $A=\{0,1\}$ and $B=\{u,v,w\}$
= **Exercise 1.6**

$$A\times B=\left\{\begin{matrix}(0,u), & (0,v), & (0,w)\\ (1,u), & (1,v), & (1,w)\end{matrix}\right\}$$

$$S\times T=\left\{(s,t) \mid \begin{matrix} s\in S \text{ and} \\ t\in T\end{matrix}\right\}$$

$A\times B$ contains $6=2\cdot 3$ elements.

If $|A|=a$ and $|B|=b$ then $|A\times B|=ab$.

Let S and T be sets.
The sets S and T are <u>equal</u> if S and T satisfy

$$S \subseteq T \text{ and } T \subseteq S.$$

Write $S = T$ if S and T are equal.

<u>Problem 1.16</u> Prove that

If A and B are sets then $A \cap B \subseteq A$.

<u>Proof</u> Assume A and B are sets.
To show: $A \cap B \subseteq A$.
To show: If $s \in A \cap B$ then $s \in A$.
Assume $s \in A \cap B$. To show: $s \in A$.
Since $s \in A \cap B$ then $s \in A$ and $s \in B$.
So $s \in A$.
So $A \cap B \subseteq A$. //.

<u>Problem 1.17</u> Prove that

$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

<u>Proof</u>: This statement is the definition of $A = B$. //.

Problem 1.24 = Exercise 1.9 Prove that

$A \subseteq B$ if and only if $A \cup B = B$.

Proof: To show: (a) If $A \subseteq B$ then $A \cup B = B$.

(b) If $A \cup B = B$ then $A \subseteq B$.

(a) Assume $A \subseteq B$. To show: $A \cup B = B$.

To show: (aa) $A \cup B \subseteq B$.

(ab) $B \subseteq A \cup B$.

(aa) To show: If $x \in A \cup B$ then $x \in B$.

Assume $x \in A \cup B$. To show: $x \in B$

Since $x \in A \cup B$ then $x \in A$ or $x \in B$.

To show: $x \in B$.

Case 1: $x \in A$.

Since $A \subseteq B$ then $x \in B$.

Case 2: $x \in B$

Then $x \in B$.

So $A \cup B \subseteq B$.

(ab) To show: $B \subseteq A \cup B$.

To show: If $y \in B$ then $y \in A \cup B$.

Assume $y \in B$. By definition of ~~y~~ $A \cup B$

then $y \in A \cup B$. So $B \subseteq A \cup B$. So $A \cup B = B$.

(b) To show: If $A \cup B = B$ then $A \subseteq B$.

Assume $A \cup B = B$. To show: $A \subseteq B$.

To show: If $x \in A$ then $x \in B$.

Assume $x \in A$. To show: $x \in B$.

Since $x \in A$ then $x \in A \cup B$.

Since $A \cup B = B$ then $x \in B$.

So $A \subseteq B$. //.

<u>Exercise 1.3</u> Prove that

if $A = B$ then $A \subseteq B$.

<u>Proof</u> Assume $A = B$. To show: $A \subseteq B$.

Since $A = B$ then $A \subseteq B$ and $B \subseteq A$.

So $A \subseteq B$. //.

<u>Exercise 1.4</u> Prove that

If $A \subseteq B$ then $A = B$. $(*)$

Counterexample:

Let $A = \{1, 2\}$ and $B = \{1, 2, 3, 4\}$

Then $A \subseteq B$ (since $1 \in B$ and $2 \in B$).

Also $B \not\subseteq A$ since $3 \in B$ and $3 \notin A$.

So $A \neq B$ since $B \not\subseteq A$.

So $(*)$ does not always hold.