27.07.2015 (1)

Reading Seminar

A. Ram

(Nonwreath)

Modified Macdonald polynomials

Hilbert Scheme

$T = \mathbb{C}^\times \times \mathbb{C}^\times$

$$K^T(pt) = \mathrm{Rep}(T) = \mathbb{Z}[q^{\pm 1}, t^{\pm 1}]$$

Theorem $D^b(\mathrm{Coh}^T(\mathrm{Hilb}_n(\mathbb{C}^2)) \cong D^b(\mathrm{Coh}^{T \times S_n}((\mathbb{C}^2)^n))$

Symmetric functions Λ

Applying K_0 gives an isomorphism

$$\bigoplus_{n \in \mathbb{Z}_{\geq 0}} K^T(\mathrm{Hilb}_n(\mathbb{C}^2))_{loc} \xrightarrow{\sim} \Lambda \quad \text{symmetric functions}$$

class of T-fixed pt. $[I_\mu] \longmapsto \tilde{H}_\mu$ modified Macdonald polynomial

The Procesi bundle

$$P_n \xrightarrow{\pi_n} \mathrm{Hilb}_n(\mathbb{C}^2)$$

and $I_\mu = T$-fixed point

$\tilde{H}_\mu = S_n\text{-}\mathrm{Fch}(\pi_n^{-1}(I_\mu))$

$U_q\mathfrak{gl}_1^{tor}$ = quantum toroidal $\mathfrak{gl}_1$

$U_q\mathfrak{gl}_1^{tor}$ acts on Λ by vertex operators

$$\tilde{D}(z) \text{ and } \tilde{D}^*(z)$$

$U_q\mathfrak{gl}_1^{tor}$ acts on $\bigoplus_{n \in \mathbb{Z}_{\geq 0}} K^T(\mathrm{Hilb}_n(\mathbb{C}^2))$ by

loop generators e_k, f_k, h_j and h_j^*.

Wreath Modified Macdonald polynomials

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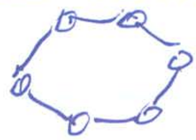
Nakajima quiver varieties for 

$\Gamma_n = S_n \ltimes (\mathbb{Z}/r\mathbb{Z})^n$ and $\beta^\vee = (\beta_1, \dots, \beta_n) \in \mathbb{Z}^n$ with $\beta_1 + \dots + \beta_n = 0$.

Theorem $D^b(\mathrm{Coh}^T(\mathcal{M}_{\beta,n})) \cong D^b(\mathrm{Coh}^{T\times\Gamma_n}((\mathbb{C}^2)^n))$

r-alphabet symmetric functions $\Lambda^{\otimes r}$

$$\bigoplus_{n \in \mathbb{Z}_{\geq 0}} K^T(\mathcal{M}_{\beta,n})_{loc} \longrightarrow \Lambda^{\otimes r}$$

$$[I_{\mu^\bullet}^{ut_{\beta^\vee}}] \longmapsto \tilde{H}_{\mu^\bullet}^{ut_{\beta^\vee}}$$

The wreath Procesi bundle

$$\begin{array}{c} P_{\beta,n} \\ \downarrow \pi \\ \mathcal{M}_{\beta,n} \end{array}$$

and $I_{\mu^\bullet}^{ut_{\beta^\vee}}$ is a T-fixed in $\mathcal{M}_{\beta,n}$

$\tilde{H}_{\mu^\bullet}^{ut_{\beta^\vee}} = \Gamma_n\text{-}\mathrm{Fch}(\pi^{-1}(I_{\mu^\bullet}^{ut_{\beta^\vee}}))$

$U_q\mathfrak{gl}_r^{tor}$ = quantum toroidal $\mathfrak{gl}_r$ is generated by

loop generators $e_i(z)$, $f_i(z)$, $\psi_i^{\pm}(z)$

or vertex operators $E^{(i)}(z)$, $F^{(i)}(z)$

Then

$$\mathcal{D}_1^{(i)}(\tilde{H}_{\mu^\bullet}^{ut_{\beta^\vee}} \otimes e^{\beta}) = A_{\mu^\bullet}^{(i)}(q^{-1}, t^{-1}) \cdot \tilde{H}_{\mu^\bullet}^{ut_{\beta^\vee}} \otimes e^{\beta}$$

$$\mathcal{D}_1^{(i)*}(\tilde{H}_{\mu^\bullet}^{ut_{\beta^\vee}} \otimes e^{\beta}) = A_{\mu^\bullet}^{(i)}(q, t) \cdot \tilde{H}_{\mu^\bullet}^{ut_{\beta^\vee}} \otimes e^{\beta}$$

eigenvalue equations.