Questions for Assignment 1

MAST90017 Representation Theory
Semester II 2015
Lecturer: Arun Ram
to be turned in on 6 August 2015 before 5pm

(1) Determine the irreducible representations, the indecomposable modules, the ideals, the radical, the center, and the traces on the algebra $A = M_{n_1}(\mathbb{C}) \oplus M_{n_2}(\mathbb{C}) \oplus M_{n_3}(\mathbb{C}) \oplus M_{n_4}(\mathbb{C})$.

(2) Determine the irreducible representations, the indecomposable representations, the ideals, the radical, the center, and the traces on the algebra A of $n \times n$ upper triangular matrices.

(3) Determine the irreducible representations, the indecomposable representations, the ideals, the radical, the center, and the traces on the algebra A of $n \times n$ strictly upper triangular matrices.

(4) Let μ_r be a cyclic group of order r and let $G_{r,r,2}$ be the dihedral group of order $2r$. Determine the irreducible representations and the indecomposable representations of the group algebras $\mathbb{C}\mu_r$ and $\mathbb{C}G_{r,r,2}$.

(5) Let p be a prime and let G be the group of upper unitriangular matrices with entries in $\mathbb{F}_p$.

  (a) Show that if $p = 2$ then G is isomorphic to the quaternion group of order 8.
  (b) Determine the irreducible representations and the indecomposable representations of the group algebra $\mathbb{C}G$.

(6) Let S_n denote the symmetric group. Determine the irreducible representations and the indecomposable representatons of the group algebras $\mathbb{C}S_2$, $\mathbb{C}S_3$ and $\mathbb{C}S_4$.

(7) Let A_n denote the alternating group. Determine the irreducible representations and the indecomposable representatons of the group algebras $\mathbb{C}A_2$, $\mathbb{C}A_3$ and $\mathbb{C}A_4$.

(8) Determine the irreducible representations and the indecomposable representatons of the group algebras $\mathbb{C}G_{r,1,2}$, for $r \in \mathbb{Z}_{>0}$.

(9) Determine the irreducible representations and the indecomposable representations of the group algebras $\mathbb{C}PGL_2(\mathbb{F}_2)$, $\mathbb{C}PGL_2(\mathbb{F}_3)$ and $\mathbb{C}PGL_2(\mathbb{F}_4)$.

(10) Determine the finite dimensional irreducible representations and the finite dimensional indecomposable representations of $U\mathfrak{sl}_2$.

(11) Determine the finite dimensional irreducible representations and the finite dimensional indecomposable representations of $U_q\mathfrak{sl}_2$.

(12) Determine the irreducible representations (including infinite dimensional irreducible representations) and the finite dimensional indecomposable representations of the algebra generated by p, q, h with relations $[p, q] = h$, $[h, p] = 0$, and $[h, q] = 0$.

(13) Determine the finite dimensional irreducible representations and the finite dimensional indecomposable representations of the algebra $\mathbb{C}[x_1, x_2]$.

(14) Determine the irreducible representations and the indecomposable representations of the algebra generated by x_1, x_2, x_3 with relations $x_i x_j = x_j x_i$ and $x_1^3 = 0$, $x_2^5 = 0$ and $x_3^4 = 0$.

(15) Determine the finite dimensional irreducible representations and the finite dimensional indecomposable representations of the algebra generated by x_1, x_2 with relations $x_1 x_2 = 0$, $x_2 x_1 = 0$, $(x_1 - 5)^{231} = 0$ and $(x_2 - 7)^{37} = 0$.

(16) Determine the finite dimensional irreducible representations and the finite dimensional indecomposable representations of the group algebras $\mathbb{C}T$, where T is the tetrahedral group.

(17) Determine the finite dimensional irreducible representations and the finite dimensional indecomposable representations of the group algebras $\mathbb{C}O$, where O is the octahedral group.

(18) Determine the finite dimensional irreducible representations and the finite dimensional indecomposable representations of the group algebras $\mathbb{C}I$, where I is the icosahedral group.

(19) Let $\mathbb{F}$ be a field. Determine the conjugacy classes of $GL_n(\mathbb{F})$.

(20) Determine the conjugacy classes of the groups $G_{r,p,n}$.