Problem Set: Graphing
620-205 Semester I 2010

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1. Graphs of the Basic Functions

(1) Graph $f(x) = |x|$.

(2) Graph $f(x) = \lfloor x \rfloor$.

(3) Graph $f(x) = 2$.

(4) Graph $f(x) = x$.

(5) Graph $f(x) = x^2$.

(6) Graph $f(x) = x^3$.

(7) Graph $f(x) = x^4$.

(8) Graph $f(x) = x^5$.

(9) Graph $f(x) = x^6$.

(10) Graph $f(x) = x^{100}$.

(11) Graph $f(x) = x^{-1}$.

(12) Graph $f(x) = x^{-2}$.

(13) Graph $f(x) = x^{-3}$.

(14) Graph $f(x) = x^{-4}$.

(15) Graph $f(x) = x^{-100}$.

(16) Graph $f(x) = e^x$.

(17) Graph $f(x) = \sin x$.

(18) Graph $f(x) = \cos x$.

(19) Graph $f(x) = \tan x$.

(20) Graph $f(x) = \cot x$.

(21) Graph $f(x) = \sec x$.

(22) Graph $f(x) = \csc x$.

(23) Graph $f(x) = \sqrt{x}$.

(24) Graph $f(x) = x^{1/3}$.

(25) Graph $f(x) = x^{1/4}$.

(26) Graph $f(x) = x^{1/5}$.

(27) Graph $f(x) = x^{1/6}$.

(28) Graph $f(x) = \dfrac{1}{\sqrt{x}}$.

(29) Graph $f(x) = x^{-1/3}$.

(30) Graph $f(x) = x^{-1/4}$.

(31) Graph $f(x) = \ln x$.

(32) Graph $f(x) = \arcsin x$.

(33) Graph $f(x) = \arccos x$.

(34) Graph $f(x) = \arctan x$.

(35) Graph $f(x) = \operatorname{arccot} x$.

(36) Graph $f(x) = \operatorname{arcsec} x$.

(37) Graph $f(x) = \operatorname{arccsc} x$.

2. Graphing Polynomials

(1) Graph $f(x) = a$, where a is a constant.

(2) Graph $f(x) = ax + b$, where a and b are constants.

(3) Graph $f(x) = a(x - c) + b$, where $a, \ , \ , b$ and c are constants.

(4) Graph $f(x) = \begin{cases} 2 - x, & \text{if } x \geq 1, \\ x, & \text{if } 0 \leq x \leq 1. \end{cases}$

(5) Graph $f(x) = \begin{cases} 2 + x, & \text{if } x > 0, \\ 2 - x, & \text{if } x \leq 0. \end{cases}$

(6) Graph $f(x) = \begin{cases} 1 - x, & \text{if } x < 1, \\ x^2 - 1, & \text{if } x \geq 1. \end{cases}$

(7) Graph $f(x) = 2x - x^2$.

(8) Graph $f(x) = x - x^2 - 27$.

(9) Graph $f(x) = 3x^2 - 2x - 1$.

(10) Graph $f(x) = x^3$.

(11) Graph $f(x) = x^3 - x + 1$.

(12) Graph $f(x) = x^3 - x - 1$.

(13) Graph $f(x) = (x - 2)^2(x - 1)$.

(14) Graph $f(x) = 2x^3 - 21x^2 + 36x - 20$.

(15) Graph $f(x) = 2x^3 + x^2 + 20x$.

(16) Graph $f(x) = 1 - x^4$.

(17) Graph $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$.

(18) Graph $f(x) = \ = 3x^4 - 16x^3 + 18x^2$.

(19) Graph $f(x) = x^5 - 4x^4 + 4x^3$.

(20) Graph $f(x) = x^3(x - 2)^2$.

(21) Graph $f(x) = (x - 2)^4(x + 1)^3(x - 1)$.

3. Graphing Rational Functions

(1) Graph $f(x) = y$, where $x^2 + y^2 = 1$.

(2) Graph $f(x) = \sqrt{1 - x^2}$.

(3) Graph $f(x) = \sqrt{a^2 - x^2}$, where a is a constant.

(4) Graph $f(x) = y$, where $(x - h)^2 + (y - k)^2 = r^2$, and h, k and r are constants.

(5) Graph $f(x) = y$, where $x^2 + y^2 - 2hx - 2ky + h^2 + k^2 = r^2$, and h, k and r are constants.

(6) Graph $f(x) = y$ where $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and a and b are constants.

(7) Graph $f(x) = y$, where $x = a\cos\theta$, $y = b\sin\theta$, and a and b are constants.

(8) Graph $f(x) = (b/a)\sqrt{a^2 - x^2}$, where a and b are constants.

(9) Graph $f(x) = y$, where $x^2 - y^2 = 1$.

(10) Graph $f(x) = y$, where $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, and a and b are constants.

(11) Graph $f(x) = ax^2 - b$, where a and b are constants.

(12) Graph $f(x) = y$, where $x = 2y^2 - 1$.

(13) Graph $f(x) = y$, where $x = \cos 2\theta$ and $y = \cos\theta$.

(14) Graph $f(x) = b\sqrt{x - a}$, where a and b are constants.

(15) Graph $f(x) = \sqrt{x + 2}$.

(16) Graph $f(x) = -\sqrt{x + 2}$.

(17) Graph $f(x) = y$, where $y^2\left(x^2 - x\right) = x^2 - 1$.

(18) Graph $f(x) = y$, where $x = \frac{y^2 - 1}{y^2 + 1}$.

(19) Graph $f(x) = \frac{\sqrt{1 + x}}{\sqrt{1 - x}}$.

(20) Graph $f(x) = \frac{x^2}{\sqrt{x + 1}}$.

(21) Graph $f(x) = x\sqrt{32 - x^2}$.

(22) Graph $f(x) = x\sqrt{1 - x^2}$.

4. Graphing sequences

(1) Graph the sequence $a_n = n$.

(2) Graph the sequence $a_n = (-1)^n n$.

(3) Graph the sequence $a_n = n^2$.

(4) Graph the sequence $a_n = 12n - n^3$.

(5) Graph the sequence $a_n = n!$.

(6) Graph the sequence $a_n = \frac{1}{n}$.

(7) Graph the sequence $a_n = 3 - \frac{1}{n}$.

(8) Graph the sequence $a_n = \frac{1}{n^p}$.

(9) Graph the sequence $a_n = \frac{1}{n!}$.

(10) Graph the sequence $a_n = \frac{n}{n(n+1)}$.

(11) Graph the sequence $a_n = \frac{1}{n} - \frac{1}{n+1}$.

(12) Graph the sequence $a_n = \frac{(-1)}{n+1}$.

(13) Graph the sequence $a_n = \frac{(-1)^{n+1}}{n}$.

(14) Graph the sequence $a_n = (-1)^n\left(1 + \frac{1}{n}\right)$.

(15) Graph the sequence $a_n = \dfrac{n}{2n+1}$.

(16) Graph the sequence $a_n = \dfrac{2n}{n+1}$.

(17) Graph the sequence $a_n = \dfrac{n}{n^2+1}$.

(18) Graph the sequence $a_n = \dfrac{3n+1}{2n+5}$.

(19) Graph the sequence $a_n = \dfrac{n^2-1}{2n^2+3}$.

(20) Graph the sequence $a_n = \dfrac{i^n}{n^2}$.

(21) Graph the sequence $a_n = \dfrac{n+2i}{n}$.

(22) Graph the sequence $a_n = \dfrac{4n+3}{4n^2+3n+1}$.

(23) Graph the sequence $a_k = \dfrac{1}{\left(3k^4-7k^2+5\right)^{\frac{1}{3}}}$.

(24) Graph the sequence $a_n = \dfrac{(n!)^2}{(2n)!}$.

(25) Graph the sequence $a_n = \dfrac{(n!)^2 5^n}{(2n)!}$.

(26) Graph the sequence $a_n = (-1)^n$.

(27) Graph the sequence $a_n = n^{1/n}$.

(28) Graph the sequence $a_n = \left(1+\dfrac{1}{n}\right)^n$.

(29) Graph the sequence $a_n = e^{in\pi/7}$.

(30) Graph the sequence $a_n = \sqrt{n}$.

(31) Graph the sequence $a_n = \dfrac{1}{\sqrt{n}}$.

(32) Graph the sequence $a_n = \sqrt{n+1} - \sqrt{n}$.

(33) Graph the sequence $a_n = \sqrt{n}\left(\sqrt{n+1} - \sqrt{n}\right)$.

(34) Let $x \in \mathbb{R}$ with $|x| < 1$. Graph the sequence $a_n = x^n$.

(35) Let $x \in \mathbb{R}$ with $x > 0$. Graph the sequence $a_n = x^{1/n}$.

(36) Let $x \in \mathbb{R}$. Graph the sequence $a_n = \left(1 + \dfrac{x}{n}\right)^n$.

(37) Let $x \in \mathbb{R}$. Graph the sequence $a_n = \dfrac{1 - x^{n+1}}{1 - x}$.

(38) Let $x \in \mathbb{R}$. Graph the sequence $a_n = 1 + x + \cdots + x^n$.

(39) Graph the sequence given by $a_1 = 3$ and $a_n = \dfrac{1}{2}\left(a_{n-1} + \dfrac{5}{a_{n-1}}\right)$.

(40) Let $a \in \mathbb{R}$ with $a > 0$. Fix a positive real number x_1. Graph the sequence given by $x_{n+1} = \dfrac{1}{2}\left(x_n + \dfrac{a}{x_n}\right)$.

(41) Let $\alpha,\ \beta \in \mathbb{R}_{>0}$. Graph the sequence given by $a_1 = \alpha$ and $a_{n+1} = \sqrt{\beta + a_n}$.

(42) Let $\alpha,\ \beta \in \mathbb{R}_{>0}$. Graph the sequence given by $a_1 = \alpha$ and $a_{n+1} = \beta + \sqrt{a_n}$.

(43) Graph the sequence given by $x_1 = 1$ and $x_{n+1} = \dfrac{1}{2 + x_n}$.

(44) Fix a real number x_1 between 0 and 1. Graph the sequence given by $x_{n+1} = \dfrac{1}{7}\left(x_n^3 + 2\right)$. Estimate the solution to $x^3 - 7x + 2 = 0$ to three decimal places and verify that the limit is a solution to the equation $x^3 - 7x + 2 = 0$.

(45) Graph the sequence given by $a_1 = 0$, $a_{2k} = \frac{1}{2}a_{2k+1}$, and $a_{2k+1} = \frac{1}{2} + a_{2k}$.

5. Graphing Other Functions

(1) Graph $f(x) = \lfloor x \rfloor$.

(2) Graph $f(x) = |x|$.

(3) Graph $f(x) = |x - 5|$.

(4) Graph $f(x) = \left|x^2 - 1\right|$.

(5) Graph $f(x) = \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{if } x = 0, \\ \text{-1}, & \text{if } x < 0. \end{cases}$

(6) Graph $f(x) = (x - 1)^{1/3}$.

(7) Graph $f(x) = x^{2/3}$.

(8) Graph $f(x) = \dfrac{1}{(x - 1)^{2/3}}$.

(9) Graph $f(x) = x(1 - x)^{2/5}$.

(10) Graph $f(x) = x^{2/3}(6 - x)^{1/3}$.

(11) Graph $f(x) = y$, where $\sqrt{x} + \sqrt{y} = 1$.

(12) Graph $f(x) = y$, where $x^{2/3} + y^{2/3} = a^{2/3}$, where a is a constant.

(13) Graph $f(x) = y$, where $x = a\cos^3\theta$ and $y = a\sin^3\theta$.

(14) Graph $f(x) = \sin x$.

(15) Graph $f(x) = \sin 2x - x$.

(16) Graph $y = \sin x - \cos x$, for $-\pi/3 < x < 0$.

(17) Graph $y(x) = 2\cos x - \sin 2x$.

(18) Graph $y = \dfrac{\sin x}{x}$.

(19) Graph $y = \sin(1/x)$.

(20) Graph $y = e^{-x}$.

(21) Graph $y = e^{1/x}$.

(22) Graph $y = e^{-x^2}$.

(23) Graph $y = \ln\left(4 - x^2\right)$.

6. Where is a Function Continuous?

(1) Graph $f(x) = x^2 + 3x + 4$. For which values of x is the function continuous?

(2) Graph

$$f(x) = \begin{cases} \dfrac{x^2 - x - 6}{x - 3}, & \text{if } x \neq 3, \\ 5, & \text{if } x = 3. \end{cases}$$

For which values of x is the function continuous?

(3) Graph

$$f(x) = \begin{cases} \dfrac{\sin 3x}{x}, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0. \end{cases}$$

For which values of x is the function continuous?

(4) Graph

$$f(x) = \begin{cases} \dfrac{1 - \cos x}{x^2}, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0. \end{cases}$$

For which values of x is the function continuous?

(5) Let $k \in \mathbb{R}$. Graph

$$f(x) = \begin{cases} \dfrac{\sin 2x}{5x}, & \text{if } x \neq 0, \\ k, & \text{if } x = 0. \end{cases}$$

For which values of k is the function continuous?

(6) Graph

$$f(x) = \begin{cases} x - 1, & \text{if } 1 \le x < 2, \\ 2x - 3, & \text{if } 2 \le x \le 3. \end{cases}$$

For which values of x is the function continuous?

(7) Graph

$$f(x) = \begin{cases} \cos x, & \text{if } x \ge 0, \\ -\cos x, & \text{if } x < 0. \end{cases}$$

For which values of x is the function continuous?

(8) Graph

$$f(x) = \begin{cases} \sin(1 / x), & \text{if } x \ne 0, \\ 0, & \text{if } x = 0. \end{cases}$$

For which values of x is the function continuous?

(9) Let $a \in \mathbb{R}$. Graph

$$f(x) = \begin{cases} a\,x + 5, & \text{if } x \le 2, \\ x - 1, & \text{if } x > 2. \end{cases}$$

For which values of x is the function continuous at $x = 2$?

(10) Graph

$$f(x) = \begin{cases} 1 + x^2, & \text{if } 0 \le x \le 1, \\ 2 - x, & \text{if } x > 1. \end{cases}$$

For which values of x is the function continuous?

(11) Graph $f(x) = 2x - |x|$. For which values of x is the function continuous?

(12) Let $a \in \mathbb{R}$. Graph

$$f(x) = \begin{cases} 2x - 1, & \text{if } x < 2, \\ a, & \text{if } x = 2, \\ x + 1, & \text{if } x > 2. \end{cases}$$

For which values of a is the function continuous?

(13) Graph

$$f(x) = \begin{cases} \dfrac{|x - a|}{x - a}, & \text{if } x \ne a, \\ 1, & \text{if } x = a. \end{cases}$$

For which values of x is the function continuous?

(14) Graph

$$f(x) = \begin{cases} \dfrac{x - |x|}{2}, & \text{if } x \neq 0, \\ 2, & \text{if } x = 0. \end{cases}$$

For which values of x is the function continuous?

(15) Graph

$$f(x) = \begin{cases} \sin x, & \text{if } x < 0, \\ x, & \text{if } x \geq 0. \end{cases}$$

For which values of x is the function continuous?

(16) Graph

$$f(x) = \begin{cases} \dfrac{x^n - 1}{x - 1}, & \text{if } x \neq 1, \\ n, & \text{if } x = 1. \end{cases}$$

For which values of x is the function continuous?

(17) Graph $f(x) = \cos x$. For which values of x is the function continuous?

(18) Graph $f(x) = \cos |x|$. For which values of x is the function continuous?

(19) Graph $f(x) = \lfloor x \rfloor$. For which values of x is the function continuous?

(20) Graph

$$f(x) = \begin{cases} x^3 - x^2 + 2x - 2, & \text{if } x \neq 1, \\ 4, & \text{if } x = 1. \end{cases}$$

For which values of x is the function continuous?

(21) Graph $f(x) = |x| + |x - 1|$, for $-1 \leq x \leq 2$. For which values of x is the function continuous?

7. Existence of Limits

(1) Graph $y = \left(\dfrac{1}{x}\right)$ and explain why $\lim\limits_{x \to 0}\left(\dfrac{1}{x}\right)$ does not exist.

(2) Graph $y = (x)$ and explain why $\lim\limits_{x \to \pi/2} \tan(x)$ does not exist.

(3) Graph $y = \sec(x)$ and explain why $\lim\limits_{x \to \pi/2} \sec(x)$ does not exist.

(4) Graph $y = \csc(x)$ and explain why $\lim\limits_{x \to 0} \csc(x)$ does not exist.

(5) Graph $y = \ln(x)$ and explain why $\lim\limits_{x \to -1} \ln(x)$ does not exist.

(6) Graph $y = \sin\left(\frac{1}{x}\right)$ and explain why $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist.

(7) Graph $y = \cos(x)$ and explain why $\lim_{x \to \infty} \cos(x)$ does not exist.

(8) Graph $y = \text{sgn}(x)$, where

$$\text{sgn}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0. \\ -1, & \text{if } x < 0 \end{cases}$$

Explain why $\lim_{x \to 0} \text{sgn}(x)$ does not exist.

(9) Graph $y = 2^{1/x}$ and explain why $\lim_{x \to 0} 2^{1/x}$ does not exist.

(10) Graph $y = 2^{1/(1-x)}$ and explain why $\lim_{x \to 1} 2^{1/(1-x)}$ does not exist.

8. References

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