Product rule for integrals

$$\int u \frac{dv}{dx}\, dx = uv - \int v \frac{du}{dx}\, dx.$$

5.17 $\int x e^{6x} dx = \int \frac{1}{6} x 6 e^{6x} dx$

$= \frac{1}{6} \int (x 6 e^{6x} + e^{6x} - e^{6x})\, dx$

$= \frac{1}{6} (x e^{6x} - \frac{1}{6} e^{6x}) + c = \frac{1}{6} x e^{6x} - \frac{1}{36} e^{6x} + c,$

where c is a constant.

5.18 $\int \log(x)\, dx = \int (\log(x) + x\frac{1}{x} - x\frac{1}{x})\, dx$

$= x\log(x) - x + c,$ where c is a constant.

5.19 $\int x^2 \log(4x)\, dx = \frac{1}{3} \int 3x^2 \log(4x)\, dx$

$= \frac{1}{3} \int (3x^2 \log(4x) + x^3 \frac{1}{4x} \cdot 4 - x^2)\, dx$

$= \frac{1}{3} (x^3 \log(4x) - \frac{1}{3} x^3) + c$

$= \frac{1}{3} x^3 \log(4x) - \frac{1}{9} x^3 + c,$ where c is a constant.

<u>Example</u> $\int \cos(x)\,dx = \sin(x) + c$, where c is a constant.

$\int (\sin(x))^4 \cos(x)\,dx = \frac{1}{5}(\sin(x))^5 + c$, where c is a constant,

since $\frac{d}{dx}\left(\frac{1}{5}(\sin(x))^5 + c\right) = \frac{1}{5}\, 5(\sin(x))^4 \cos(x) + 0$.

<u>Example</u> $\int \sin(x)^4 \cos(x)^3\,dx$

$= \int \sin(x)^4 \cos(x)^2 \cos(x)\,dx$

$= \int \sin(x)^4 (1 - \sin(x)^2) \cos(x)\,dx$

$= \int (\sin(x)^4 - \sin(x)^6) \cos(x)\,dx$

$= \frac{1}{5}\sin(x)^5 - \frac{1}{7}\sin(x)^7 + c$, where c is a constant.

<u>5.15</u> $\int \sin(x)^{25} \cos(x)^5\,dx$

$= \int \sin(x)^{25} \cos(x)^4 \cos(x)\,dx$

$= \int \sin(x)^{25} ((\cos(x))^2)^2 \cos(x)\,dx.$

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$$= \int \sin(x)^{25} \left(1 - \sin(x)^2\right)^2 \cos(x)\,dx$$

$$= \int \sin(x)^{25} \left(1 - 2\sin(x)^2 + \sin(x)^4\right) \cos(x)\,dx$$

$$= \int \left(\sin(x)^{25} - 2\sin(x)^{27} + \sin(x)^{29}\right) \cos(x)\,dx$$

$$= \frac{1}{26}\sin(x)^{26} - \frac{2}{28}\sin(x)^{28} + \frac{1}{30}\sin(x)^{30} + c$$

where c is a constant.

5.14

$$\int \sin(x)^4\,dx = \int \left(\frac{e^{ix} - e^{-ix}}{2i}\right)^4 dx$$

$$= \int \frac{1}{2^4 i^4}\left((e^{ix})^4 + 4(e^{ix})^3(-e^{-ix}) + 6(e^{ix})^2(-e^{-ix})^2 + 4e^{ix}(-e^{-ix})^3 + (-e^{-ix})^4 \right) dx$$

$$= \int \frac{1}{16}\left(e^{i4x} - 4e^{i2x} + 6 - 4(e^{-i2x}) + e^{-i4x}\right) dx$$

$$= \int \frac{1}{16}\left(2\frac{(e^{i4x} + e^{-i4x})}{2} - 2\cdot 4\left(\frac{e^{i2x} + e^{-i2x}}{2}\right) + 6\right) dx$$

$$= \int \frac{1}{16}\left(2\cos(4x) - 8\cos(2x) + 6\right) dx$$

$$= \frac{1}{16}\left(2\frac{\sin(4x)}{4} - 8\frac{\sin(2x)}{2} + 6x\right) + c$$

$$= \frac{1}{32}\sin(4x) - \frac{1}{4}\sin(2x) + \frac{3}{8}x + c,$$ where c is a constant

5.20 $\int x^5(x^3+1)^{1/2}dx = \int x^2 \cdot x^3(x^3+1)^{1/2}dx$

$= \int x^2(x^3+1-1)(x^3+1)^{1/2}dx$

$= \int x^2\left((x^3+1)^{3/2} - (x^3+1)^{1/2}\right)dx$

$= \frac{1}{3}\int\left((x^3+1)^{3/2} - (x^3+1)^{1/2}\right)3x^2\,dx$

$= \frac{1}{3}\left(\frac{2}{5}(x^3+1)^{5/2} - \frac{2}{3}(x^3+1)^{3/2}\right) + c.$

$= \frac{2}{15}(x^3+1)^{5/2} - \frac{2}{9}(x^3+1)^{3/2} + c$, where c is a constant.

5.27 and 5.31

$\int \frac{3x+1}{x^2+4x+4}dx = \int \frac{3x+1}{(x+2)^2}dx = \int \frac{3(x+2)-5}{(x+2)^2}dx$

$= \int\left(\frac{3}{x+2} - \frac{5}{(x+2)^2}\right)dx = 3\int\frac{1}{x+2}dx - 5\int(x+2)^{-2}dx$

$= 3\log(x+2) - 5\cdot\frac{1}{-1}(x+2)^{-1} + c$

$= 3\log(x+2) + \frac{5}{x+2} + c$, where c is a constant.

2.48 4^{th} roots of 1 are

$$\{z \in \mathbb{C} \mid z^4 = 1\} = \{1, i, -1, i^3\} = \{e^0, e^{i\pi/2}, e^{i\pi}, e^{i3\pi/2}\}$$

$$= \{e^0, e^{i6\pi/12}, e^{i12\pi/12}, e^{i18\pi/12}\}$$

The number $2-2i\sqrt{3}$ has

length $|2-2i\sqrt{3}| = \sqrt{2^2+(-2\sqrt{3})^2} = \sqrt{4+12} = 4.$

and

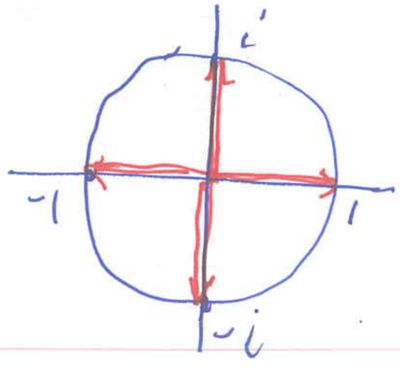
$$2-2i\sqrt{3} = 4\left(\frac{2}{4} - \frac{2\sqrt{3}}{4}i\right) = 4\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 4e^{-i\pi/3}$$

so $4^{1/4}\left(e^{-i\pi/3}\right)^{1/4} = \sqrt{2}\,e^{-i\pi/12}$ is a 4^{th} root of $2-2i\sqrt{3}$

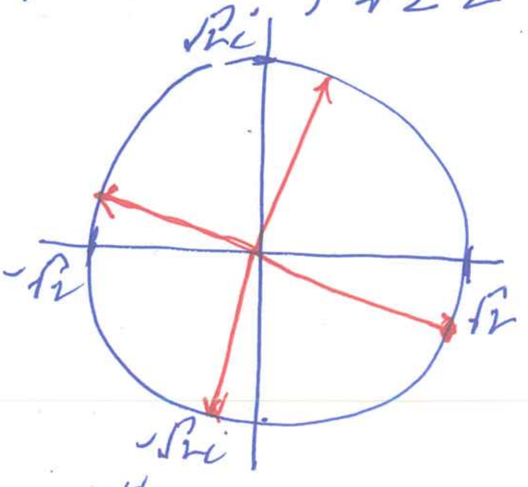
The 4^{th} roots of $2-2i\sqrt{3}$ are

$$\sqrt{2}\,e^{-i\pi/12}\cdot 1^{1/4} = \left\{\begin{array}{l} \sqrt{2}\,e^{-i\pi/12}\cdot e^0,\ \sqrt{2}\,e^{-i\pi/12}\cdot e^{i6\pi/12}, \\ \sqrt{2}\,e^{-i\pi/12}\,e^{i12\pi/12},\ \sqrt{2}\,e^{-i\pi/12}\,e^{i18\pi/12} \end{array}\right\}$$

$$= \{\sqrt{2}\,e^{-i\pi/12}, \sqrt{2}\,e^{i5\pi/12}, \sqrt{2}\,e^{i11\pi/12}, \sqrt{2}\,e^{i17\pi/12}\}$$



4^{th} roots of 1



4^{th} roots of $2-2i\sqrt{3}$.