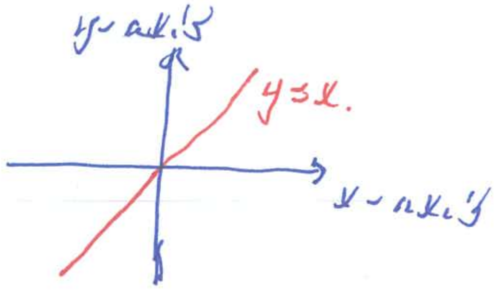
The identity function $id_S : S \to S$ is given by $id_S(s) = s$.

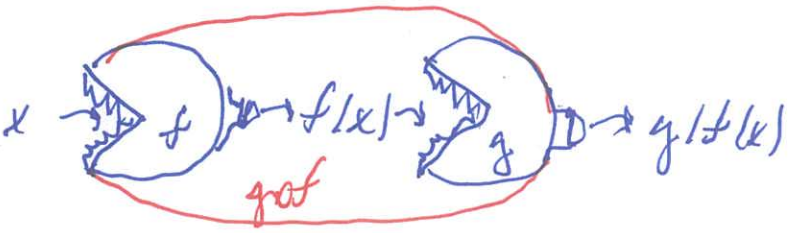
$$id_{\mathbb{R}} : \mathbb{R} \to \mathbb{R}, \quad x \mapsto x$$ has



Composition

$$X \xrightarrow{f} Y \xrightarrow{g} Z, \quad x \mapsto f(x) \mapsto g(f(x))$$ is $$g\circ f : X \to Z, \quad x \mapsto g(f(x))$$

$g\circ f$ does not make sense unless the codomain of f is the domain of g.



Example $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto x^2$ and $g : \mathbb{R} \to \mathbb{R}$, $x \mapsto e^x$

has $(f\circ g)(x) = f(g(x)) = f(e^x) = (e^x)^2 = e^{2x}$

and $(g\circ f)(x) = g(f(x)) = g(x^2) = e^{x^2}$.

So $(f\circ g)(1) = e^{2\cdot 1} = e^2 \approx 9$ and $(g\circ f)(1) = e^{1^2} = e^1 \approx 3$.

the <u>inverse function</u> to $f: S \to M$ is

a function $g: M \to S$ such that

$g \circ f = id_S$ and $f \circ g = id_M$.

i.e. $g(f(x)) = x$ and $f(g(m)) = m$.

<u>theorem</u> The inverse function to $f: S \to M$ exists (is a function)

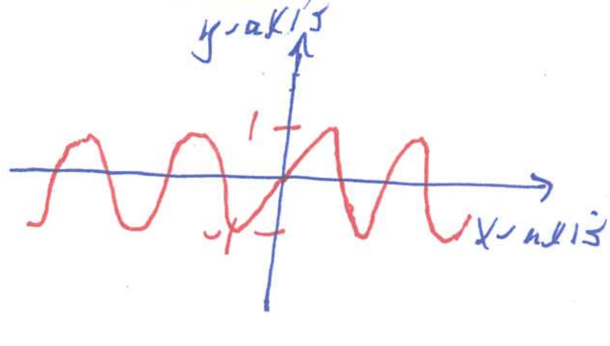
if and only if

$f: S \to M$ is bijective.

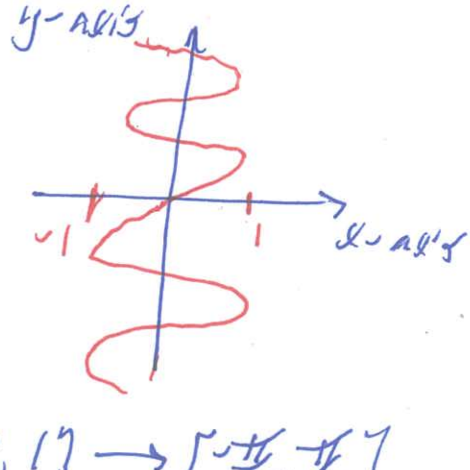
<u>3.27</u> $h: \mathbb{R} \to \mathbb{R}$, $x \mapsto e^x$ does <u>not</u> have an inverse function

$f: \mathbb{R} \to \mathbb{R}_{>0}$, $x \mapsto e^x$ has inverse function $g: \mathbb{R}_{>0} \to \mathbb{R}$, $x \mapsto \log(x)$

$\mathbb{R} \to \mathbb{R}$, $x \mapsto \sin(x)$ does <u>not</u> have an inverse.



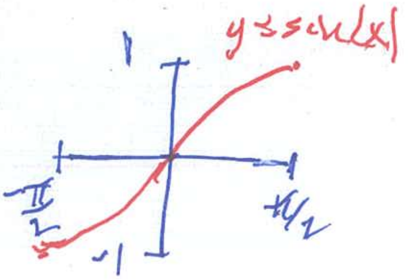
should have "inverse"



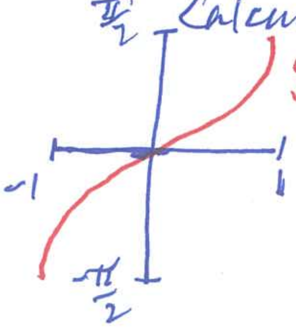
$[-\frac{\pi}{2}, \frac{\pi}{2}] \to [-1, 1]$, $x \mapsto \sin(x)$ has inverse $[-1, 1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$, $x \mapsto \arcsin(x)$.

and these undo each other,

$\sin(\arcsin(x)) = x$ and $\arcsin(\sin(x)) = x$



has inverse



$y = \arcsin(x)$ is $\sin(y) = x$.

<u>Examples 3.33 and 3.34</u>

With this arcsin function

$$\arcsin(\tfrac{1}{2}) = \arcsin(\sin(\tfrac{\pi}{3})) = \tfrac{\pi}{3}.$$

$$\arcsin(\sin(\tfrac{7\pi}{4})) = \arcsin(\sin(\tfrac{-\pi}{4})) = -\tfrac{\pi}{4}$$

<u>Image and range</u>

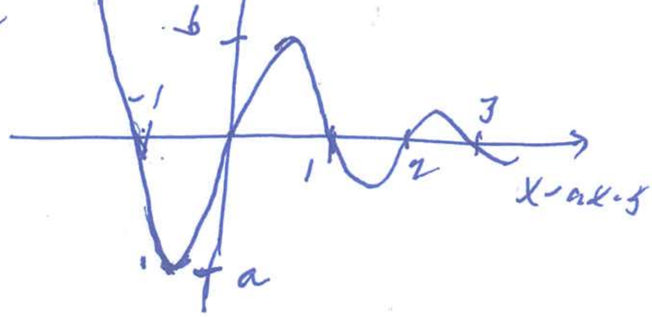
Let $f: S \to M$ be a function. Let $A \subseteq S$:

$$f(A) = \{f(a) \mid a \in A\}$$

The <u>image</u> of f, or <u>range</u> of f, is

$$f(S) = \{f(s) \mid s \in S\}$$

<u>Example 3.7</u> $f: \mathbb{R} \to \mathbb{R}$ with



has

$$f([0,1]) = [0,b]$$

$$f((-\infty,-1)) = (0,\infty)$$

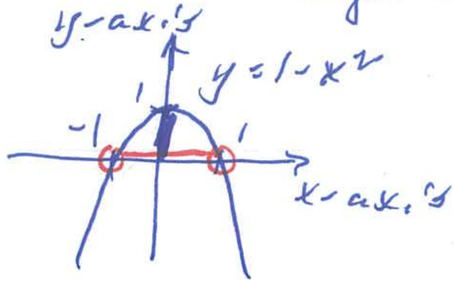
$$f([-1,2]) = [a,b]$$

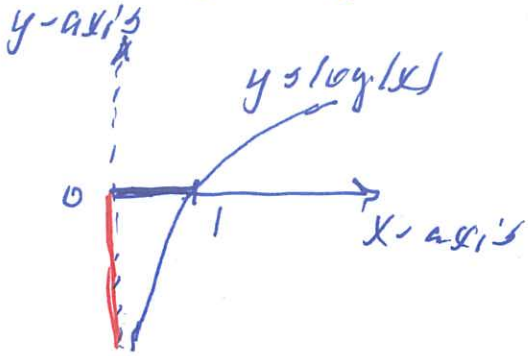
and $\mathrm{Im}(f) = f(\mathbb{R}) = [a,\infty)$

Source and target (implied domain and codomain)

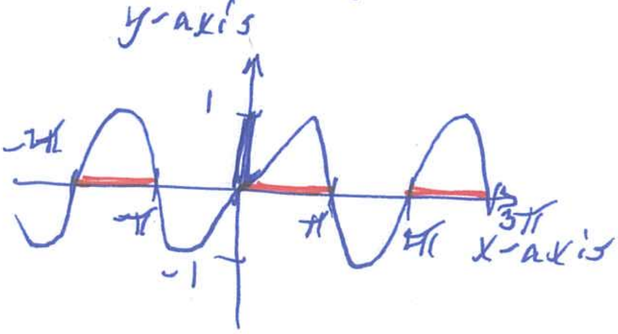
Find the largest $S \subseteq \mathbb{R}$ and the smallest $M \subseteq \mathbb{R}$ such that $f : S \to M$ is a function.

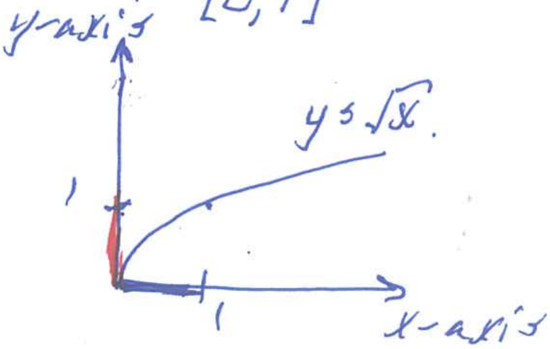
3.35 $f : S \longrightarrow M$, $x \mapsto \log(1-x^2)$ has $S = (-1,1)$, $M = (-\infty, 0]$





3.36a $f : S \longrightarrow M$, $x \mapsto \sqrt{\sin(x)}$ has $S = \bigcup_{k \in \mathbb{Z}} [2k\pi, (2k+1)\pi]$, $M = [0,1]$





3.36b $f : S \to M$, $x \mapsto \sqrt{\cos(x) - 2}$ has $S = \emptyset$, $M = \emptyset$