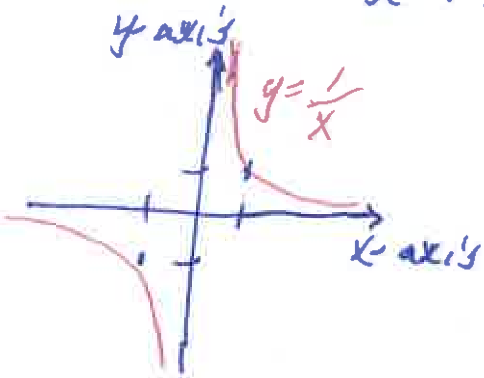
3.10 and 3.19

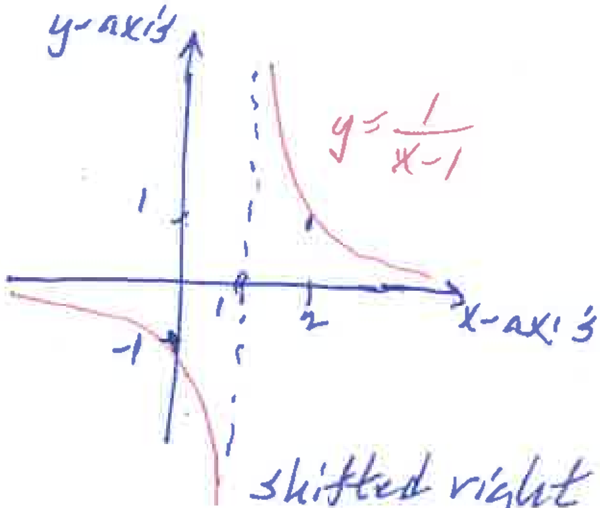
$g: \mathbb{R}\setminus\{1\} \to \mathbb{R}$ given by $g(x) = \dfrac{2x-7}{1-x}$

has $g(x) = -\dfrac{(2x-7)}{(x-1)} = -\dfrac{(2(x-1)-5)}{(x-1)} = -\left(2 - \dfrac{5}{x-1}\right)$

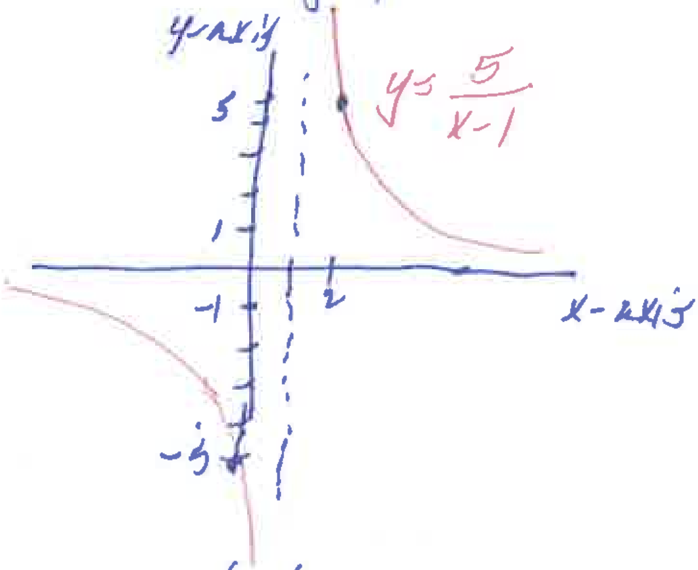
$= -2 + \dfrac{5}{x-1}.$



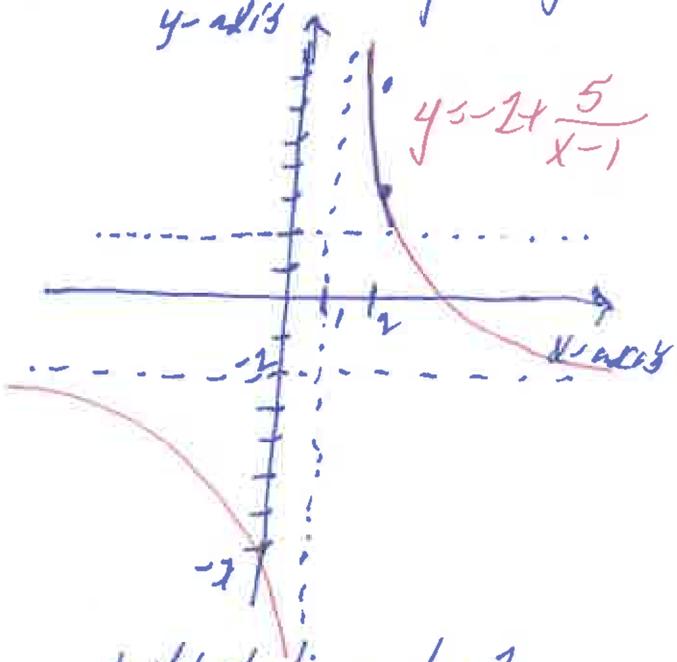
Basic graph



shifted right by 1



scaled



shifted down by -2

Then

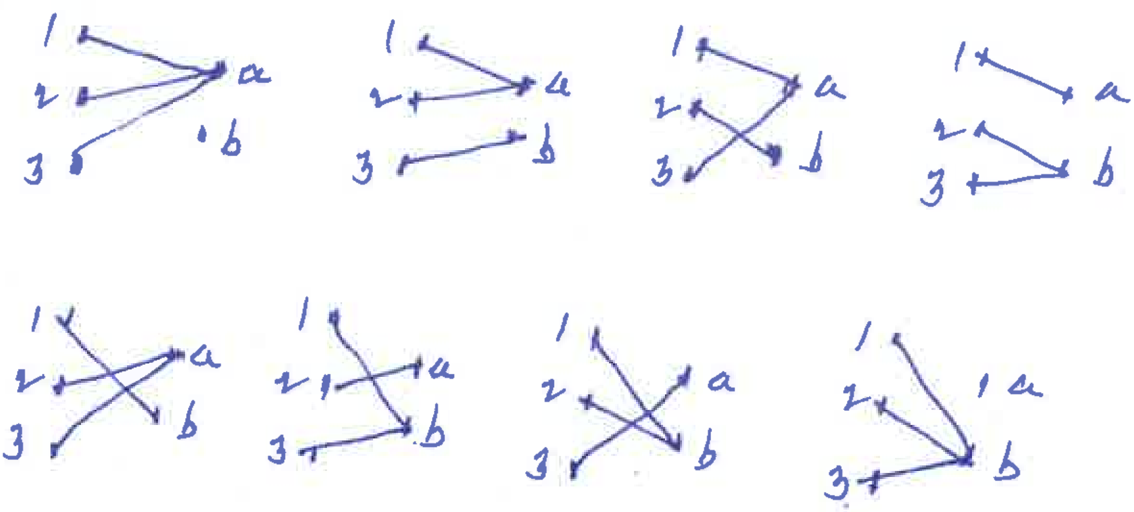
g is injective

g is <u>not</u> surjective (-2 doesn't occur as a mark)

g is <u>not</u> bijective.

But $h:\mathbb{R}\setminus\{1\}\to\mathbb{R}\setminus\{-2\}$ given by $h(x)=\dfrac{2x-1}{1-x}$ is injective, is surjective and is bijective.

<u>3.18</u> Functions $f:\{1,2,3\}\to\{a,b\}$



There are $2^3=8$ of these functions.

<u>None</u> of them are injective

<u>None</u> of them are bijective

All but the first and last are surjective.

<u>3.17</u> $f:\{0,1,2\}\to\{0,1,2\}$ given by

$$f(x)=\begin{cases} x+1, & \text{if } x\neq 2\\ 0, & \text{if } x=2\end{cases}$$

has

$$\begin{array}{rcl} f:\{0,1,2\} & \to & \{0,1,2\}\\ 0 & \longmapsto & 1\\ 1 & \longmapsto & 2\\ 2 & \longmapsto & 0\end{array}$$

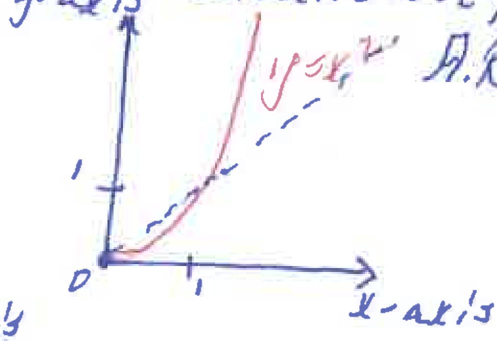
and $\Gamma_f=\{(0,1),(1,2),(2,0)\}$

f is injective,

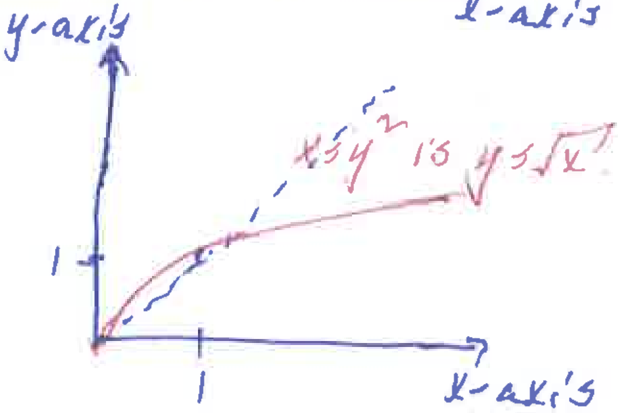
f is surjective, and

f is bijective.

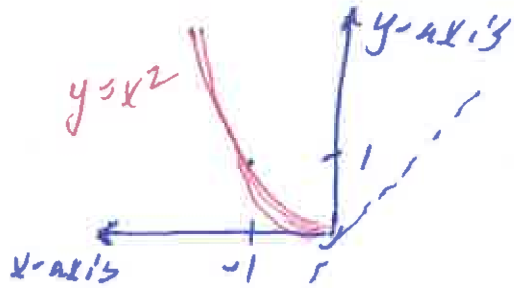
<u>3.22</u> $f: \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, $x \mapsto x^2$ is bijective



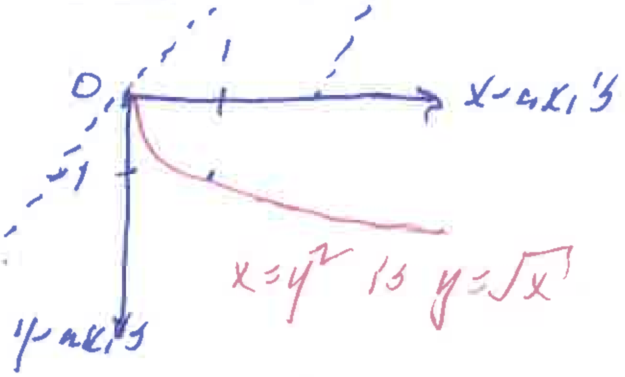
with inverse $g: \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, $x \mapsto \sqrt{x}$



$k: \mathbb{R}_{\leq 0} \to \mathbb{R}_{\geq 0}$, $x \mapsto x^2$ is bijective

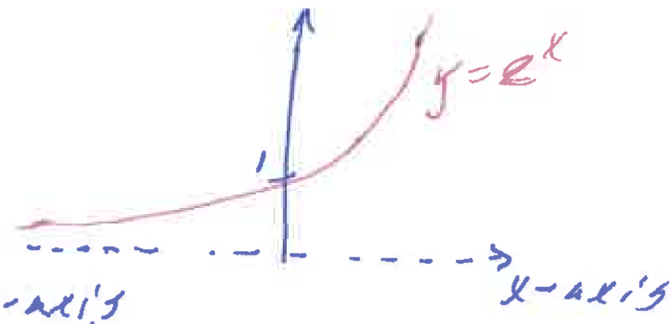


with inverse $k: \mathbb{R}_{\geq 0} \to \mathbb{R}_{\leq 0}$, $x \mapsto \sqrt{x}$

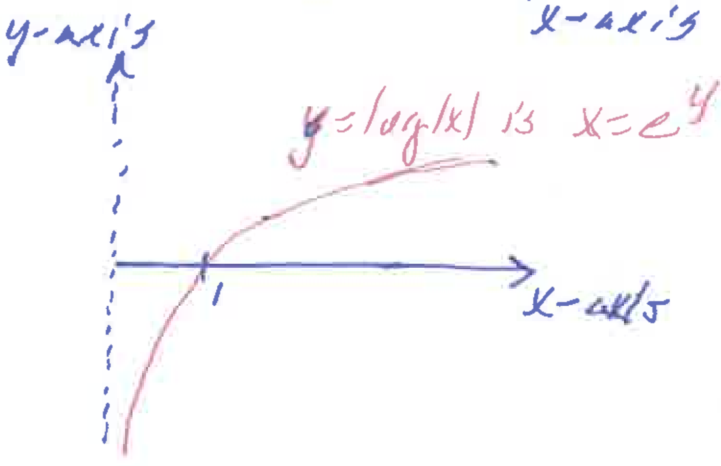


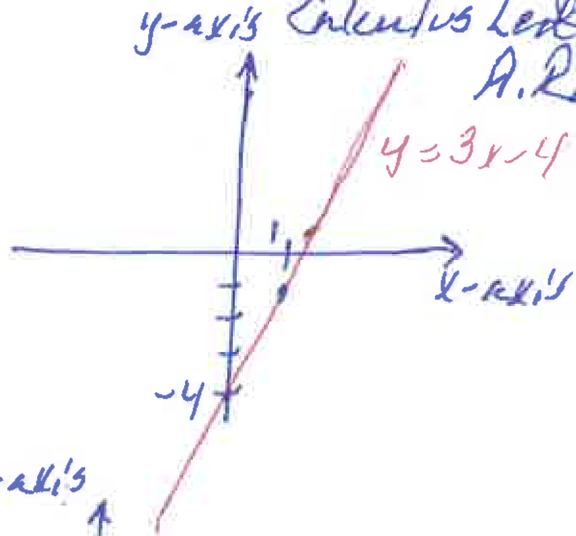
<u>3.24b</u>

$\mathbb{R} \to \mathbb{R}_{>0}$, $x \mapsto e^x$ is bijective



with inverse $\mathbb{R}_{>0} \to \mathbb{R}$, $x \mapsto \log(x)$

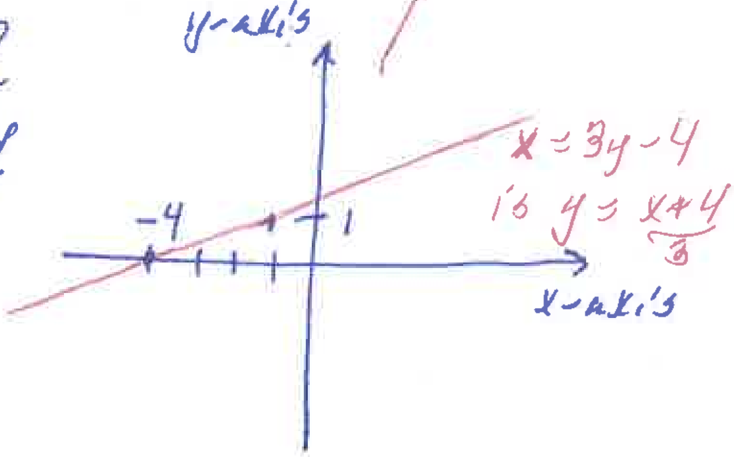




<u>3.26a</u>

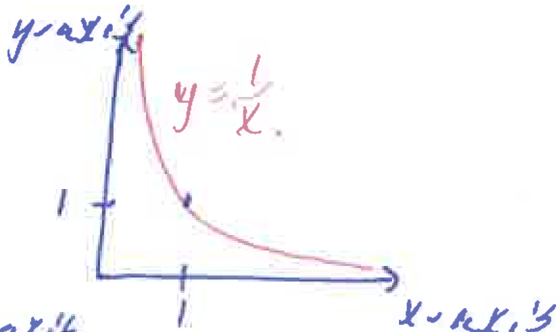
$$f: \mathbb{R} \to \mathbb{R} \quad \text{is bijective}$$
$$x \mapsto 3x-4$$

with inverse $g: \mathbb{R} \to \mathbb{R}$
$$x \mapsto \frac{x+4}{3}$$

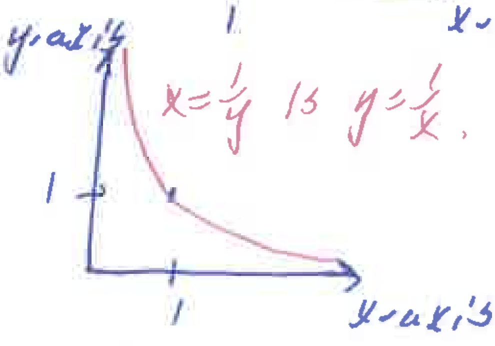


<u>3.21b</u>

$$f: \mathbb{R}_{>0} \to \mathbb{R}_{>0} \quad \text{is bijective}$$
$$x \mapsto \frac{1}{x}$$



with inverse $g: \mathbb{R}_{>0} \to \mathbb{R}_{>0}$
$$x \mapsto \frac{1}{x}$$



<u>3.30</u> $f: \mathbb{C} \to \mathbb{C}$ is bijective
$$z \mapsto \bar{z}$$

with inverse $g: \mathbb{C} \to \mathbb{C}$,
$$z \mapsto \bar{z}$$

since $\bar{\bar{z}} = g(f(z)) = z$ and $\bar{\bar{z}} = f(g(z)) = z$.