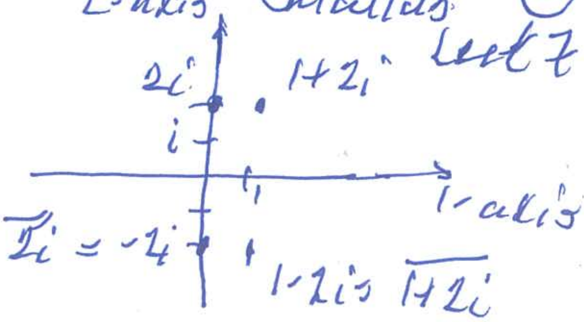
19.03.2025
Calculus
Lect 7
(1)

Graphing



2.2 $\{x\in\mathbb{C} \mid x^2-2x+5=0\}$

The quadratic formula gives

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4\cdot1\cdot5}}{2\cdot1}=\frac{2\pm\sqrt{4-20}}{2}=\frac{2\pm\sqrt{16(-1)}}{2}$$
$$=\frac{2\pm4i}{2}=1\pm2i$$

2.1 $\{x\in\mathbb{C} \mid x^2+4=0\}=\{2i,-2i\}$ since

$(2i)^2+4=4i^2+4=-4+4=0$ and

$(-2i)^2+4=4i^2+4=-4+4=0.$

2.13 $\{z\in\mathbb{C} \mid z^2-6z+10=0\}=\{3+i,3-i\}.$

2.11 $\bar{z}$ is reflection of z in the 1-axis.

Length, or modulus

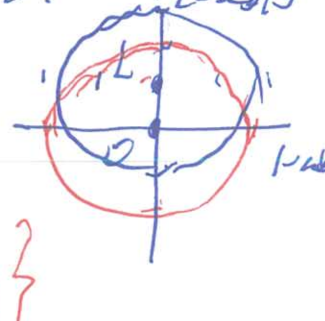
$|z|$ = length of z = distance from z to 0.

$|z-i|$ = length of $z-i$ = distance from z to i.

$|z-3-2i|=|z-(3+2i)|$ = length of $z-(3+2i)$

= distance from z to $3+2i$.

2.37



$\{z\in\mathbb{C} \mid |z-i|=2\}$

$=\{z\in\mathbb{C} \mid$ distance from z to i is $2\}$

$\{z\in\mathbb{C} \mid |z|=2\}$

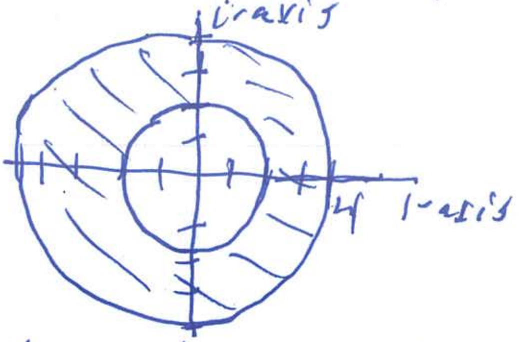
$=\{z\in\mathbb{C} \mid$ distance from z to 0 is $2\}$

<u>2.38</u> $\{z \in \mathbb{C} \mid |z-3-2i| < 3\}$

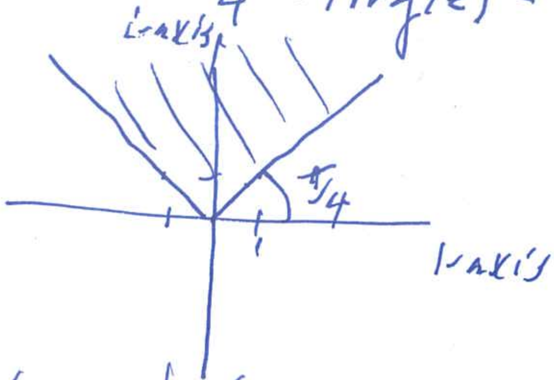
$= \{z \in \mathbb{C} \mid |z-(3+2i)| < 3\}$

$= \left\{ z \in \mathbb{C} \;\middle|\; \begin{array}{l} \text{distance from} \\ z \text{ to } 3+2i \\ \text{is } < 3 \end{array} \right\}$

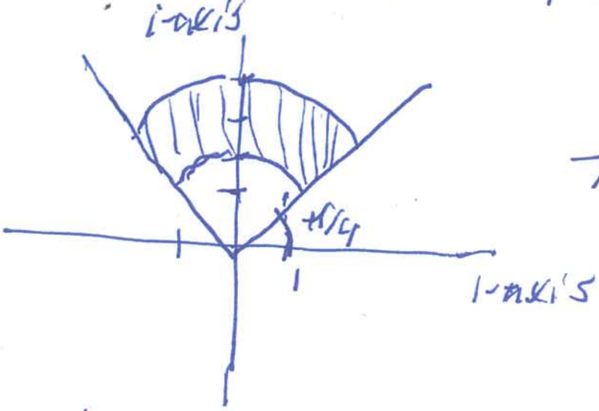
<u>2.39</u> $A = \{z \in \mathbb{C} \mid 2 \le |z| \le 4 \text{ and } \frac{\pi}{4} < \operatorname{Arg}(z) < \frac{3\pi}{4}\}$



$\{z \in \mathbb{C} \mid 2 \le |z| \le 4\}$



$\{z \in \mathbb{C} \mid \frac{\pi}{4} < \operatorname{Arg}(z) < \frac{3\pi}{4}\}$



The set A

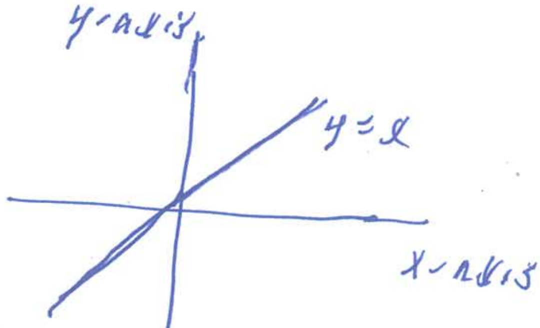
<u>2.41</u> $\{z \in \mathbb{C} \mid z = i\bar{z}\}$

$= \{x+iy \in \mathbb{C} \mid x+iy = i(\overline{x+iy})\}$

$= \{x+iy \in \mathbb{C} \mid x+iy = i(x-iy)\}$

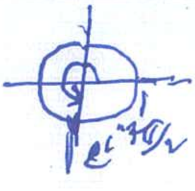
$= \{x+iy \in \mathbb{C} \mid x+iy = y+ix\}$

$= \{x+iy \in \mathbb{C} \mid x = y\}$



The set $\{z \in \mathbb{C} \mid z = i\bar{z}\}$.

2.46 $i^7 = i\cdot i\cdot i\cdot i\cdot i\cdot i\cdot i = (-1)\cdot(-1)\cdot(-1)\cdot i = -i.$



$$i^7 = \left(e^{i\frac{\pi}{2}}\right)^7 = e^{i\frac{7\pi}{2}} = e^{i(2\pi+\frac{3}{2}\pi)} = e^{i\frac{3}{2}\pi} = -i.$$

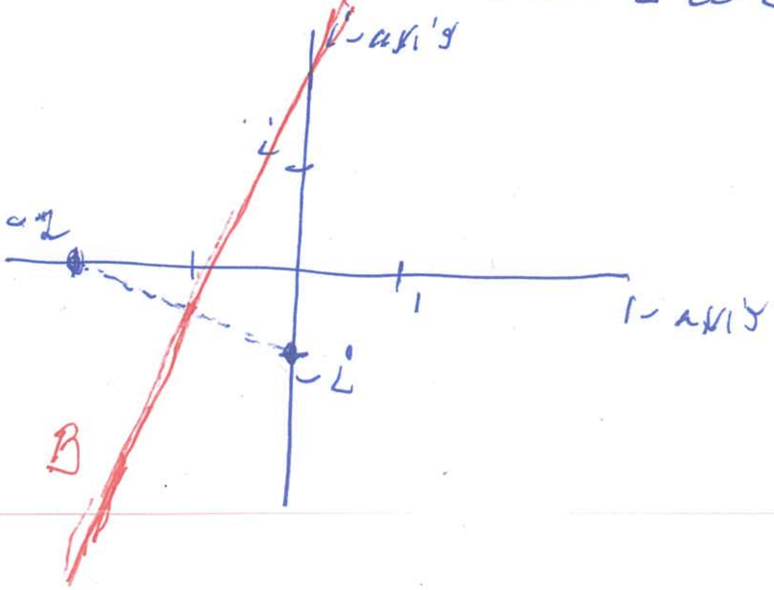
2.27
$$5e^{i\frac{3\pi}{4}} = 5\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right) = -\frac{5\sqrt{2}}{2}+\frac{5\sqrt{2}}{2}i$$

has

$$\mathrm{Re}\left(5e^{i\frac{3\pi}{4}}\right) = -\frac{5\sqrt{2}}{2}, \quad \mathrm{Im}\left(5e^{i\frac{3\pi}{4}}\right) = \frac{5\sqrt{2}}{2}$$

$$\left|-\frac{5\sqrt{2}}{2}+\frac{5\sqrt{2}}{2}i\right| = 5, \quad \mathrm{Arg}\left(-\frac{5\sqrt{2}}{2}+\frac{5\sqrt{2}}{2}i\right) = \frac{3\pi}{4}.$$

2.40
$$B = \{z\in\mathbb{C} \mid |z+2| = |z+i|\}$$
$$= \{z\in\mathbb{C} \mid |z-(-2)| = |z-(-i)|\}$$
$$= \left\{z\in\mathbb{C} \;\middle|\; \begin{array}{c}\text{distance from } z \text{ to } -2 \\ \text{equals} \\ \text{distance from } z \text{ to } -i\end{array}\right\}$$



19.03.2025 (4)

Calculus Lect 7

A. Ram

$z = re^{i\theta}$

$z = x + yi$