14.03.2025 (1)
Calculus Lect. 5
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Contrapositive

If $\underline{A}$ then $\underline{B}$

has contrapositive

If $\underline{\text{not } B}$ then $\underline{\text{not } A}$

Example 1.32

(a) If $A \subseteq B$ then $A \cup B = B$

has contrapositive

If $A \cup B \neq B$ then $A \not\subseteq B$.

(b) If $n \in \mathbb{Z}_{>0}$ is divisible by 4 then n is even

has contrapositive

If $n \in \mathbb{Z}_{>0}$ and n is odd then n is not divisible by 4.

If and only if

Example 1.34 Let A and B be sets. Assume $A \neq \emptyset$ and $B \neq \emptyset$. Prove that

$A \subseteq B$ if and only if $A \cup B = B$.

Solution:

To show: (a) If $A \subseteq B$ then $A \cup B = B$

(b) If $A \cup B = B$ then $A \subseteq B$.

(a) To show: If $A \cup B \neq B$ then $A \not\subseteq B$.

Assume $A \cup B \neq B$.

To show: There exists $a \in A$ such that $a \notin B$.

Since $A \cup B \neq B$ then there exists $c \in A \cup B$ such that $c \notin B$.

Since $c \in A$ or $c \in B$ and $c \notin B$ then $c \in A$.

So there exists $c \in A$ such that $c \notin B$.

So $A \not\subseteq B$.

(b) To show: If $A \cup B = B$ then $A \subseteq B$

Assume $A \cup B = B$

To show: $A \subseteq B$.

To show: If $a \in A$ then $a \in B$.

Assume $a \in A$

To show: $a \in B$

Since $a \in A$ then $a \in A \cup B$.

Since $A \cup B = B$ then $a \in B$.

So $a \in B$

So $A \subseteq B$. //

~~Some~~ All mathematicians know the abbreviations

$\exists$ for 'there exists'

$\forall$ for 'if'

These notation make the exposition less readable and often alienate readers.

Another one of these is

$\therefore$ for 'so'

Example 1.29 Assume

$A = \{4n \mid n \in \mathbb{Z}\}$ and

$B = \{2m+2 \mid m \in \mathbb{Z}\}$.

Prove that $A \subseteq B$.

Solution

To show: If $a \in A$ then $a \in B$.

Assume $a \in A$.

Since $a \in A$ then there exists $n \in \mathbb{Z}$ such that $a = 4n$

Since $a = 4n$ then

$$a = 4n = 4n - 2 + 2 = 2(2n-1) + 2.$$

To show: $a \in B$.

To show: There exists $m \in \mathbb{Z}$ such that $a = 2m+2$.

Let $m = 2n - 1$.

Then $a = 2m+2$.

So $a \in B$. So $A \subseteq B$

Example 1.29 Assume

$A = \{4n \mid n \in \mathbb{Z}\}$ and $B = \{2m+2 \mid m \in \mathbb{Z}\}$

Prove that $A \subseteq B$.

Solution

To show: $\forall a \in A$, $a \in B$.

Assume $a \in A$.

Since $a \in A$, $\exists n \in \mathbb{Z} \mid a = 4n$.

Since $a = 4n$, $a = 4n = 4n - 2 + 2$

$= 2(2n-1) + 2$.

To show: $a \in B$.

To show: $\exists m \in \mathbb{Z} \mid a = 2m+2$.

Let $m = 2n-1$. Then $a = 2m+2$.

$\therefore a \in B \therefore A \subseteq B$.