12.03.2025

Calculus Lect 4

A. Ram

If $x, y, z \in \mathbb{R}$ and $x < y$ and $y < z$ then $x < z$.

If $a, x, y \in \mathbb{R}$ and $x < y$ then $a + x < a + y$

If $a, x, y \in \mathbb{R}$ and $x < y$ and $a > 0$ then $ax < ay$.

Example 1.23

$$A = \{x \in \mathbb{R} \mid -2 - \tfrac{1}{2}x > -4\}$$

$$= \{x \in \mathbb{R} \mid -2 - \tfrac{1}{2}x + 2 > -4 + 2\}$$

$$= \{x \in \mathbb{R} \mid -\tfrac{1}{2}x > -2\}$$

$$= \{x \in \mathbb{R} \mid -\tfrac{1}{2}x + \tfrac{1}{2}x > -2 + \tfrac{1}{2}x\}$$

$$= \{x \in \mathbb{R} \mid 0 > -2 + \tfrac{1}{2}x\}$$

$$= \{x \in \mathbb{R} \mid 0 + 2 > -2 + \tfrac{1}{2}x + 2\}$$

$$= \{x \in \mathbb{R} \mid 2 > \tfrac{1}{2}x\}$$

$$= \{x \in \mathbb{R} \mid 2 \cdot 2 > 2 \cdot \tfrac{1}{2}x\}$$

$$= \{x \in \mathbb{R} \mid 4 > x\} = \cancel{(4, \infty)}\ (-\infty, 4).$$

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Example 1.24

$A = \{x \in \mathbb{R} \mid 1 - x < 3x + 2\}$

$= \{x \in \mathbb{R} \mid -1 + 1 - x < -1 + 3x + 2\}$

$= \{x \in \mathbb{R} \mid -x < 3x + 1\}$

$= \{x \in \mathbb{R} \mid x - x < x + 3x + 1\}$

$= \{x \in \mathbb{R} \mid 0 < 4x + 1\}$

$= \{x \in \mathbb{R} \mid -1 + 0 < 4x + 1 - 1\}$

$= \{x \in \mathbb{R} \mid -1 < 4x\}.$

$= \{x \in \mathbb{R} \mid \frac{1}{4} \cdot (-1) < \frac{1}{4} 4x\}$

$= \{x \in \mathbb{R} \mid -\frac{1}{4} < x\}$

$= (-\frac{1}{4}, \infty).$

Example 1.28

Prove that if $a, b, x, y \in \mathbb{R}$ and $x < y$ and $a < b$ then $x + a < y + b$.

Solution: Assume $a, b, x, y \in \mathbb{R}$ and $x < y$ and $a < b$.

To show: $x + a < y + b$.

$x + a < x + b$ (by the additive property of $a < b$ then $x + a < x + b$)

$< y + b$ (by the additive property if $x < y$ then $x + b < y + b$)

So, by transitivity, $x + a < y + b$. //

$A \subseteq B$ if A and B satisfy
if $a \in A$ then $a \in B$.

Example 1.29 Let

$A = \{4n \mid n \in \mathbb{Z}\}$ and

$B = \{2m+2 \mid m \in \mathbb{Z}\}$.

Prove that $A \subseteq B$.

Solution:

To show: If $a \in A$ then $a \in B$.

Assume $a \in A$

To show: $a \in B$.

Since $a \in A$ then there exists
$n \in \mathbb{Z}$ such that $a = 4n$.

Since $a = 4n$ then

$$\begin{aligned} a &= 4n \\ &= 4n - 2 + 2 \\ &= 2(2n-1) + 2. \end{aligned}$$

Since $n \in \mathbb{Z}$ then $2n - 1 \in \mathbb{Z}$.

So $a \in B$

So $A \subseteq B$.

Example 1.38 Let

$A = \{3n+1 \mid n \in \mathbb{Z}\}$ and

$B = \{6m+1 \mid m \in \mathbb{Z}\}$.

Show that $A \subseteq B$ is false.

Solution:

To show: $A \not\subseteq B$

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To show: there exists $a \in A$ such that $a \notin B$.

Let $a = 3 \cdot 1 + 1$. Then $a \in A$.

To show: $a \notin B$.

$a = 4 = 3 + 1 = 6 \cdot \frac{1}{2} + 1$.

Since $\frac{1}{2} \notin \mathbb{Z}$ then $a \notin B$.

So $A \not\subseteq B$.

//.