# Calculus: Student information

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**Abstract**

At the end of 2024 I decided to start to clean my office, following 40 years of career work. I had *lots* of notes on calculus teaching. It is evident that I had invested myself in my teaching over the years and I had mounds of notes, records and resources that I had developed and collected. I began to go through this, collate it, curate it, and to try to decide what might be helpful to others and what should just be dumped in the garbage. This manuscript is a transcription of information distributed to students at the commencement of the course Math 221: Calculus and Analytic Geometry in 2006.

***Key words***— Calculus

Dear Math 221 Lecture 4 and Lecture 5 students,

Welcome! To get started, I typed up a bunch of information about how I think about this course. Like everything I say, take it or leave it, but most importantly, **question it**. As you should do in dealing with any "expert", <span style="color:crimson">**question everything I say, don't ever trust me, and always get a second (and a third and a fourth) opinion.**</span> Please read, thoroughly,

- The course home page
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## Your TA

**Your TAs role.** Your TA is part of colorpurpleyour "getting through calculus" team (along with your friends, your mother, etc., etc.). This is a tough course. Your TA will help you. Help them help you by telling them where you are confused, which step you got lost, what you are feeling, how they can help. Sometimes explaining things is not so easy–help your TA explain it to you by asking questions. Asking lots of random questions helps your TA figure out exactly which point is confusing for you.

It helps to mentally put yourself in your TA's shoes. Your TA was an undergraduate just a couple of months ago. They went to the most awesome graduation party ever. They took the summer off to hang out at the pool and schmooze with the babes. The last thing they did was math, particularly, the summer right before they enter math graduate school where the professors, the older graduate students and their calculus students are going to eat them alive. They moved to graduate school a couple of weeks ago, they had to sign a lease, pay a zillion dollar security deposit (their parents no

longer give them ANY money at all) they don't get their first check until October 1, and they had to buy a car, and find the grocery store, and cook dinner for themselves (the spaghetti they made was sauce from a jar and was horrible) and at this moment this whole graduate school thing is not looking very wonderful.

The first day of class, they have no idea what they should do, they are going to be faced with you, 20 creatures staring at them with beady eyes, waiting for that moment to pounce on something they say or do to consign them to the land of "TAs the students hate" until the end of time. Yeah, you'd know what to say if you were in that situation too.

*You can help break down the wall.* Tell your TA about yourself, where you came from, who your girlfriend is and how awesome she is, what an amazing party you went to last weekend. Make CDs for your TA to listen to, cut out cartoons from the newspaper for your TA, show your TA that you are real person and not just a creature that hates them viscerally. When it comes to math, ask your TA questions, help your TA help you. Actually, your TA knows a huge amount of math, it's just that it's hard to make it come out – for some reason your TA finds it hard to make calculus come out in wonderful explanations with beautiful clarity when they frightened to the depths of their soul and about to piss their pants. Go figure. ... Give your TA a chance, if you help them, they'll help you. Back to contents

# Hints for doing calculus:

1. **Write each line with clarity and completeness**, it makes the mathematics easier.

2. **Figure out why things are done the way they are.** There is always a good easy reason. The difficulty is to think about it long enough to realise what the reason is. If you don't know and don't think of the reason then ask. Ask many different people until you get an answer that is satisfying. And then tell everybody else. Once you figure something out spread the word. It's lots easier if everyone works together.

3. **Calculus is easy.** Most of it is a no brainer. DON'T THINK, JUST COMPUTE. Unfortunately, mathematical culture loves to pretend that math is hard (it makes us feel good about ourselves) and so we have, built into our math world, that we say how hard it is LOTS of the time. But it's not true, actually math is pretty easy stuff.

4. **Calculus is cool.** Why is it cool? What makes something cool? Fine wine is cool, fancy people with lots of money spend zillions of dollars on this drink of rotten grapes that tastes terrible. Fine wine is cool, and has no applications. Ludacris is cool. Zillions of people buy these albums of noise and bad words–music that has no applications. But Ludacris is DEFINITELY cool. **Why is calculus cool?**

   ○ an answer that is too often given: calculus is cool because it has applications to other fields (biology, neuroscience, security and codes, stocks and bonds, are some of the oft quoted applications). This is equivalent to saying that calculus is cool because is is connected to other things that are cool. This is true, but it is not the only reason (or even the primary reason) math is cool.

   ○ calculus is cool because the thought process is used in every step of every day life and it gives you an edge over other people. In other words, calculus is cool because it makes you smart.

- ○ calculus is cool because cool people do it. Check out the web pages of some famous mathematicians and find out about their outside interests. Prof. Ram spends his time traveling the world, hanging out in coffee shops and bookstores and art museums, having dinner at fancy restaurants. Of course this means that you also have to be cool to do calculus.

- ○ calculus is cool because the ideas are beautiful and captivating, like a fine painting. In other words, mathematics stimulates the artist in your soul, and so doing mathematics is like listening to a CD of your favourite songs in the whole world.

5. **The hardest thing** about this course is to be honest with yourself. Really, did you stay up too late last night to be able to process properly what Prof. Ram is saying? Or did you miss that important point because your mind wandered to those darn split ends that make your hair frizzy at the ends? Can you really do that problem on the spot if you have to? Did you really study 6 hours last night or was alot of it taken up with Bob coming in to ask you about ancient Sonic Youth lyrics? Does Prof. Ram make you so nervous that you just want to turn and run? Are you or are you not actually comfortable with this cute shirt that you think is kinda cool cuz it's low cut? What is it, really, that makes you choose the seat in the very back row? The most wonderful thing is that no one else has to know these things that have you a little off kilter, but the most dangerous thing is to lie to yourself and not recognize them at face value. Be honest with yourself! Back to contents

# Big things for improving your grade.

**Writing mathematics really carefully, clearly and in complete sentences.** A very important part of this course is learning how to write mathematics. The main goal in writing is that your reader understands easily what they read. Some things about math writing:

- Math writing has alot of words in it. For examples, watch how many words Prof. Ram writes in lecture. Go to the math library on floor B2 of Van Vleck and look at a math journal and see for yourself that when mathematicians explain things to each other they use mostly words and not so many equations. Actually the symbols that we use are just abbreviations for the words–we use them because we are lazy (for example, 2+2=4 is an abbreviation for two plus two is four).

- Math writing is in complete sentences. If it is not in complete sentences and not grammatically correct then it is usually impossible to read and make sense out of. Perhaps this is why you are confused in section. If your TA is not writing complete sentences on the board in section and you are confused then tell them. We all get lazy sometimes, but you can keep your TA on track so that you understand what they are doing. Note: = is a verb. A sentence without a verb is not grammatically correct. Note: = is not the only verb.

- Prof. Ram remembers very few formulas (but is pretty good at calculus–go figure) and so to make your homework and exams understandable to Prof. Ram you will need write solutions to the problems that do not assume that the grader remembers formulas.

One of the goals of this course is to teach you how to write mathematics well, and so, **yes, we will count off if your answer is not written up properly with good grammar and good mathematical writing style.**

**Simplifying.** A very important part of mathematics (and this course) is simplifying your answers. How do you know when you have simplified sufficiently? Simplification is an aesthetic. It is a form of beauty. We like answers that are "pretty"; we don't like ugly answers. You must simplify until the

answer cannot be made prettier. Learning to simplify well requires two things: facility with algebraic manipulations and a sense of when an expression looks nice. Everybody has some internal sense of when an expression looks nicer than another but, as with any art form, to get good at it, you must practice and refine this sense. One of the goals of this course is to teach you how to simplify well, and so, **yes, we will count off if your answer is not simplified.**

## Using your resources

One of the tricks in life is to use your resources well.

1. the book assigned for the course which is available in the bookstore,
2. the calculator that you bought for college that you won't need for the homeworks and are not allowed to use on the exams for this course,
3. the lectures,
4. the discussion sections,
5. the notes that you take in class,
6. the lecture notes on the web,
7. the sample exams on the web,

there are

1. computer labs with math packages on the computers,
2. zillions of calculus books available as hand-me-downs from other students, siblings, friends, neighbors, calculus TAs, calculus professors,
3. zillions of calculus books in the UW library
4. zillions of calculus books in the used bookstores around town
5. at least 10 official tutoring services on campus
6. over 100 graduate students in the math department willing to help once in a while (this doesn't include grad. students in physics, chemistry, ...)
7. over 70 faculty in the math department willing to help once in a while (this doesn't include faculty in physics, chemistry, engineering, ...)
8. over 6000 students on campus that have taken this class before and are willing to help once in a while
9. 17 discussion sections for this class that you could go to once in a while,
10. a second lecture of this class that you are welcome to come to,
11. at least 5 other lectures of Math 221 taught by other faculty that probably explain it better than Prof. Ram does
12. at least 40 other discussion sections for those other lectures of Math 221 that are going on this term.
13. sample exams in the math library
14. lecture notes of at least 1000 other students on campus that took math221 from Prof. Ram
15. lots of calculus information and help available on the web (do a google search)
16. zillions of tutors that will help you even more if you pay them

## The book

The textbook for the course is Thomas' Calculus, **Eleventh edition**, Addison-Wesley. It is not in my blood to follow the book closely in class. What we will cover in class IS covered in the book and the syllabus indicates how the sections in the book correspond to what we are covering in class. I am most able to explain how I think about the material and how I do calculus. There is no "right way". In class I try to teach the way that I think about things. If the book happens to do some particular part of the material differently from the way I have presented it in class and you would like me to explain to you how the book does it, please do not hesitate to ask me in office hours or to make an appointment. I will do my best to explain it in as many different ways as you need to see it in order to understand it fully.

## Exams

The midterm exams are 10 problems each, verbatim from the homework, chosen randomly.

**Why the exams are easy.** The exams are taken verbatim from the homework. There is never a problem on the exam that the students have not seen before on the homework. The disadvantage is that there is lots of homework. However, effort spent on the homework problems usually translates to good scores on the exams and students that do most of the homework usually feel that they have learned a lot at the end of the course. It is quite a bit of work and requires discipline but the pay off is significant.

**Why are the midterm questions chosen randomly?** One method of assessing whether somebody knows something is to pick a random question from that subject and pose that question. If the person can answer that question then they know that subject and if not they don't, or at least they don't know it well enough. If you pay attention in daily life, and keep track, you will be amazed at how often this principle of assessment is applied. This is the idea that led to randomly choosing the problems.

**Another reason the problems are randomly chosen:** Inevitably, a tired student, looking at all the homework left to be done, will look through and think to themselves, "Prof. Ram will never put this problem on the midterm, it takes 25 minutes to do" (or any one of an infinite number of other reasons), and then not do it. As soon as they don't do that problem they don't learn that stuff. In order to curtail this chain of events the exam problems are chosen randomly.

**Why are the midterms 10 problems?** For years I adjusted the length of the midterms by doing them myself and then multiplying the time it takes me to do it by 3. A 50min exam would take me 16-17min. I found that the length of the midterms almost always turned out be around 10 problems. Finally, since it makes several other things easier (grading, dealing with student questions on how long the exam will be, since the grades are curved anyway) I decided just to fix it at 10 problems on each midterm and it has worked well.

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## Homeworks

**Amount of time the HW takes.** This is a 5 credit course. The general university guideline is that students should work on the class outside of lecture and discussion for approximately 2-3 hours per week per credit hour. This means that for this course the students should spend about 10-15 hours outside of class per week and this is the reason the homeworks are designed to take 10-15 hours. From experience we do pretty well at hitting this number on average.

**How much of the HW do we do for you?** The Homework assignments are roughly 100 problems per week. Each week

- **approximately 20 get done in lecture as examples.** In lecture I do not say "HW problem xx" I say "Example" but these examples are taken verbatim from the Homework assignments

- **approximately 10 get done in discussion section.**

- **approximately 20 get done in my office hours.** My office hours are Sunday 1:30-4:30pm in B239. They are one of your best resources for this course.

All together we do about 50 problems for you. This means that the homework assignments are really only about 50 problems long and those other problems are similar to problems that were done in class or section or office hours.

**Why do we do about half of the homework problems for you?** Shouldn't you do the homework through yourself–won't you learn it better that way? I strongly believe that it is easier to learn how to do something if someone shows you how to do it first–it is not usually very efficient if you have to figure it out from scratch. So I have no qualms about doing some of the problems for you and showing you how its done. I want you to learn how to do it. And there are enough homework problems left for you to do by yourself after I've done 50 of them for you. Back to contents

**Another reason for the long HW assignments.** Like with any skill, you can't just do it once or twice and be any good at it (have you ever met anyone that has been on a skateboard 2 times and is good at skateboarding?). You have to practice, and do it over and over. I want you to be able to DO calculus. I want you to be able to do calculus in a way that nobody can deny that you can do calculus.

**Yet another reason why:** So that all the examples I do in class are on the HW. When I lecture I always give examples to illustrate what I am trying to explain. I found, from experience, that often students were thinking about something else (the girlfriend they just broke up with, or what Oprah said, or something) and the examples I did in class weren't really sinking in. When working on a homework problem, sometimes students didn't even realize that I had done a similar example in class. Then I had a revelation– I could put all the in class examples on the homework. It worked like magic. All of a sudden the students started to care about and pay attention to the examples I did in class.

**Why are there wrong answers on the HW answer sheets?** To make the answer sheets for the homeworks Prof. Ram worked late into the night doing the homeworks. Of course Prof. Ram always does the problems perfectly;) but sometimes he makes a typing error when he is copying the answer into the answer sheet file. So the reason there are mistakes in the answer sheets is human error. Over the years we corrected the answer sheets until they were 99% correct. This is enough that you should be able to develop some self confidence for determining when you are right and the answer sheet is wrong. I want you to be self confident in your calculus abilities. Don't ever assume that something is correct just because it is printed fancy.

## Planning your time

It is more or less impossible to do the average homework assignment for this course in one night. Planning 3-4 hours per day for 3-4 days per week is one possible way for a student to manage the time on this course. I am aware that the students also have other classes to study for that will also require 2-3 hours of outside of class study per hour of class time. If you are a student are spending more than 15 calculus focused hours per week on the homework for this class please come see me and let's talk about it. If you do not keep me informed I cannot help.

- **Homework time.** If the homework takes 15 hours, this means that you need to spend 2 hours per day, 7 days a week. (It's not so different from going to workout.) If you miss a day then

you'll have to make it up the next day and put in 4 hours. It should average out to getting about 14 problems done per day.

- **Study groups.** Study groups help alot but they also take alot of time because another requirement of study groups is that you blow off alot of time talking about other things (Sex and the City, Prof. Ram's clothes, OC, OMG Do you know what happened last night...) On average, study groups make the calculus (a) easier, by a factor of 2, and (b) take more time, by a factor of 4-6. Thus if you plan to work on calculus only in study groups you should plan about 20-30 hours a week of study group for calculus.

- **Non Math221 time.** How does your daily/weekly schedule look? How much time is spent where? Did you watch TV today? Did you put it in their daily schedule. Did you call their mom today? Did you put it on your schedule? How much time did it take from your calculus time? Did you sleep enough? Will you be in good enough mental shape from so little sleep to do calculus or will the calculus take twice as long because you are tired and thinking only 1/2 as well? Back to contents

## Grades

Grades are normalised 8% for HW, 20% for MT1, 20% for MT2, 20% for MT3, 32% for the final exam.

- **How much is the homework worth?** Do the calculation: Missing 10 points on each exam is equivalent to getting a 0 on all the homeworks. So don't freak out about homework grades. On the other hand if you don't do the homework you will almost certainly not do well on the exams, especially since the exam questions are taken verbatim from the homework.

- **Computation of the grades:** At the end of the term after the final exams are graded, all the points are added up (after normalizing to the correct percentages) and a histogram is made. The general guidelines for giving the grades are 20% A, 30% B, 30% C, 20%D-F.

ANY ESTIMATE OF A GRADE THAT IS MADE BEFORE THE FINAL EXAMS ARE GRADED IS ONLY AN ESTIMATE AND MIGHT BE WAY OFF THE MARK. In particular, by experience we have noticed that there are two groups of students

1. those that don't concentrate on calculus in the first month, fail the first exam, realise that they are in trouble, and start working on calculus and do very well on the the second midterm,

2. Those that work hard on calculus in the first month and do well on the first midterm and then get cocky and start slacking and fail the second exam. Not everybody lies in one of these two groups, but these two groups are larger than you might think.

Midterm letter grades should be taken with a grain of salt. MUCH more important towards getting a good grade is to keep your point total high. Back to contents

## Why do we use a different definition of derivative than other classes?

**The problem that started it all**. When I began teaching calculus I soon realised that there was a difficulty: the students are not secure enough with their algebra skills to easily learn the calculus (which is easy) without getting bogged down with algebra difficulties (algebra is also easy, it's learning the two at the same time, in combination, that is the problem). I had

**Two options:**

- I could complain about the high schools not preparing our students well and say that it wasn't my fault that the students couldn't learn calculus well,

or

- I could teach them algebra and get their algebra skills up to the right level myself (I need students to be able to do operations like

  - Simplify $(x^2 + 2x + 1)/(x + 1)$ quickly and correctly,
  - Multiply out $(x^3 + 3x^2 + 2x + 7)(x - 5)(x^2 + 3)$ quickly and correctly
  - Simplify $\sqrt{((x^2 + 2)^3 + 7)^7}/(x^4 - 4x^2 + 11)^{2/3}$ quickly and correctly,
  - Graph $y = x^7$ quickly and correctly

  in order to make the calculus easy to learn.)

I chose this second option. The difficulty is that one cannot, realistically, take the first 4 weeks of Math 221, devote them to algebra skills, and then succeed in completing the usual Math 221 syllabus before the end of the term.

**The solution** I came up with is to define the derivative as a creature that eats functions and spits out functions, splits up sums, and satisfies the product rule. All of these operations are algebraic, and so we focus on algebra, taking derivatives, learning the algebra properties of derivatives (chain rule, derivatives of $e^x$, trig functions, etc) for the first 5 weeks.

**From a mathematical perspective I liked this:** In fact, to generalise the derivative it is standard to define it this way and consider the derivative as a derivation so that one is able to define derivatives for algebraic varieties or more general rings or function spaces. For the first 5 weeks or so I can work quite well in the field $\mathbb{Q}((x))$ where most of the functions I want to consider live and then start thinking about the, more complicated, ring of functions on $\mathbb{R}$ in week 6. $\mathbb{Q}((x))$ is the field of fractions of the formal power series ring $\mathbb{Q}[[x]]$, which is an easy completion of the easy ring $\mathbb{Q}[x]$, whereas, the ring of functions on $\mathbb{R}$ (which is not even a field and so a bit messy from this perspective) depends heavily on the structure of $\mathbb{R}$, which is a (not as easy) completion of $\mathbb{Q}$, the field of fractions of the easy ring $\mathbb{Z}$. It makes sense, pedagogically, that we should introduce the easier things first and then move on to the more complicated systems.

**The difficulty with this approach turns out to be a social one.** Though the students absorb the mathematics in this order quite easily there is significant resistance, because this is not "the way that everyone else does it" (and therefore an assumption that it can't be as good as "the usual" system). However, the more I think about it the more I'm convinced that this ordering is more sensible pedagogically: the formula

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

is simply packed with more mathematical subltety than the formula

$$\frac{d(fg)}{dx} = \frac{df}{dx}g + f\frac{dg}{dx}$$

and so it is harder to swallow.

**We still cover all the material.** The fact that the ordering of events is changed a bit DOES NOT MEAN that we don't cover the formula

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

(and explain it thoroughly in both concept and application) in this course. We do, it just comes in week 6 instead of in week 1. All ways of thinking about the derivative are useful (not just these two: for example as a slope, or as the coefficient of $t$ in $f(x+t)$) and we try to be thorough and cover them all.

## Social injustice

This course is designed for students that have not taken calculus before. It is a fact of the numbers that many students take calculus in high school, take the placement exam, place out of Math 221, and take Math 221 anyway first term freshman year (some students do not even bother taking the placement exam they just retake Math 221 anyway). Of course it makes sense, this is the first term at college, they are scared, unsure of their background and the transition, and they want a bit of buffer–the idea is that they have had calculus before and so Math 221 will be "easy".

OOPS. There are two problems:

1. **It's still pretty challenging:** Math 221 in college is not a cakewalk, especially if you have had calculus before. In Math 221 you must AVOID learning formulas and techniques, you MUST learn why everything is true, what the concept is, how to derive it AND how to explain the concept to others, AND how to write it down correctly, clearly, so it is readable, legible and understandable. Most high school math programs will not provide this training for you, and so you will have to unlearn and then relearn the material. It is the unlearning part that is the most difficult. There are also many new things to learn in Math 221 that often were not learned before college. Things like:

    ○ Why do we care about the rational numbers? What is the definition of $\pi$?
    ○ What are the real numbers?
    ○ Why is $\sin(\pi/6) = 1/2$?
    ○ Why is $\sqrt{x}$ not a function?
    ○ Why is the area of a circle $\pi r^2$?

2. **A social problem** is created by the fact that many students have had calculus in high school. It makes those students that have not had calculus in high school feel at a disadvantage and this insecurity is not healthy. The students that have had calculus in high school are cocky and this false security is dangerous.

In fact, it is the students that have had calculus before that will run into problems:

○ it will take a while before they realise that they can't be cocky, there is alot to learn, and that they have to do the work, and, by this time they will be behind and struggling to catch up. Once they do realise that they are not ahead they might be demoralized and depressed and this emotional difficulty is also dangerous. Really it is not so bad, and everybody gets through it.

○ they will have to unlearn many of the things they thought they "learned" in high school. More precisely, they will have to fill in the gaps in their knowledge, and learn the reasons why all the things they learned in high school are true and when they can use the things they learned in high school and when they can't. This is confusing because you have to somehow reorganise and add to all the stuff that is already in your head (those of us that moved recently know how much work it is to reorganise (clean up, throw things out, put what you want to keep into boxes, take it to college, and unpack it all, and put it in order along with all the new things you needs for college) the last 18 years of your life. It is easier not have all that stuff in your head to reorganize and just get it all the first time in the most recent, current, release.

**About Professor Ram**

The best place to find out about Professor Ram is the "Ask me a question" time before class. Ask me anything. I dare you.