1.22 Fundamental theorems of change and calculus

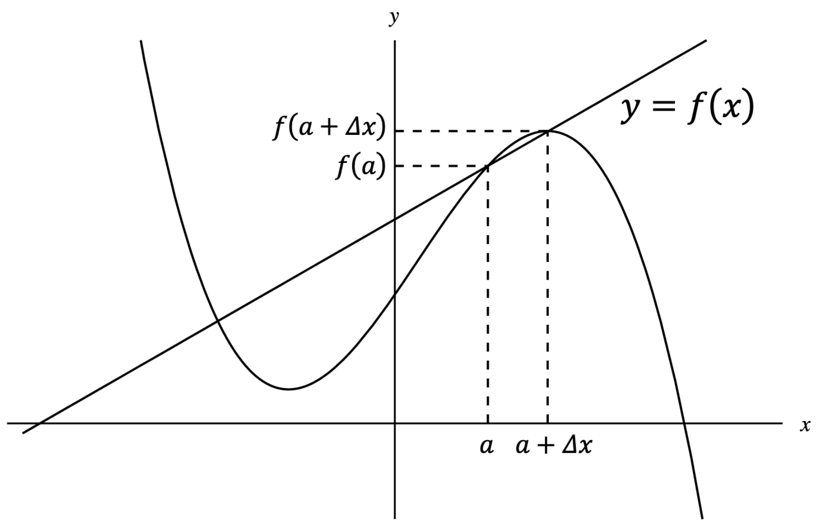
If
$$\frac{df}{dx} = g$$
then define
$$\frac{df}{dx}\Big]_{x=a} = g(a) \qquad \text{and} \qquad \left(\int g\,dx\right)\Big]_{x=a}^{x=b} = f(b) - f(a).$$

Fundamental theorem of change.
$$\frac{df}{dx}\Big]_{x=a} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}.$$

Fundamental theorem of calculus.
$$\left(\int g\,dx\right)\Big]_{x=a}^{x=b} = \lim_{N\to\infty} \left(g(a)\tfrac{1}{N} + g(a+\tfrac{1}{N})\tfrac{1}{N} + \cdots + g(b-\tfrac{1}{N})\tfrac{1}{N}\right).$$

1.23 The fundamental theorem of change

Think about
$$\frac{df}{dx}\Big]_{x=a} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$
in terms of the graph



The slope of $f(x)$ at $x = a$

$$\begin{aligned}\frac{f(a+\Delta x)-f(a)}{\Delta x} &= \frac{\text{change in } f}{\text{change in } x} \\ &= \frac{\text{rise}}{\text{run}} \\ &= \text{slope of line connecting } (a, f(a)) \text{ and } (a+\Delta x, f(a+\Delta x)).\end{aligned}$$

This gives that
$$\lim_{\Delta x\to 0} \frac{f(a+\Delta x)-f(a)}{\Delta x} = (\text{slope of } f \text{ at the point } x = a).$$

A function is *differentiable at* $x = a$ if the graph of $f(x)$ at $x = a$ exists.