

The generators and relations for In Galois Field Math(D)

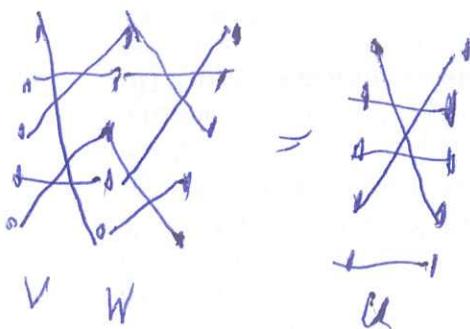
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Generators A! Permutation matrices

$$\begin{pmatrix} D & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & D & 1 & 0 \\ 0 & D & 1 & 0 & D \end{pmatrix} = \cancel{\text{matrix}}$$

Relations of $VW = U$ is given by matrix multiplication



Gamblers $\mathcal{S} = \{s_1, \dots, s_{N+1}\}$

Relations B $s_j^n = 1$, $s_j s_k = s_k s_j$ if $k \notin \{j-1, j+1\}$
 and $s_i s_{i+1} s_i = s_{i+1} s_i s_i$.

Generators B in terms of Generators A

$S_L =$
 $\text{for } i \in \{1, \dots, n\}$

Relations between relations A

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②

$$s_L^{-n} = \begin{array}{c} \overbrace{\text{---}}^n \\ \left\{ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \end{array} \quad \begin{array}{c} \text{---} \\ \left\{ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \end{array}$$

$$\begin{array}{c} \cancel{\text{---}} \\ \left\{ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \end{array} \circ \begin{array}{c} \text{---} \\ \left\{ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \end{array} = 1.$$

$$s_J s_K s_{J \cap K}^{-1} = \begin{array}{c} \text{---} \\ \left\{ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \end{array} \circ \begin{array}{c} \text{---} \\ \left\{ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \end{array} \circ \begin{array}{c} \text{---} \\ \left\{ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \end{array} = \begin{array}{c} \text{---} \\ \left\{ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \end{array} \circ \begin{array}{c} \text{---} \\ \left\{ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \end{array} = s_K s_J$$

$$s_{J \cup K} s_L s_{J \cap K}^{-1} = \begin{array}{c} \text{---} \\ \left\{ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \end{array} \circ \begin{array}{c} \text{---} \\ \left\{ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \end{array} = \begin{array}{c} \text{---} \\ \left\{ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \end{array} \quad \text{and}$$

$$s_{J \cup K} s_L s_{J \cup K}^{-1} = \begin{array}{c} \text{---} \\ \left\{ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \end{array} \circ \begin{array}{c} \text{---} \\ \left\{ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \end{array} = \begin{array}{c} \text{---} \\ \left\{ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\} \end{array} \quad \text{So } s_{J \cup K} \text{ is } s_{J \cup K}^{-1}.$$

Generators A from Generators B (the subject of)
reduced words

Let $w \in S_n$

A reduced word for w is an expression

$$w = s_{i_1} \cdots s_{i_l} \quad \text{with } i_1, \dots, i_l \in \{1, \dots, n\}$$

such that ℓ is minimal. The length of w is $\ell(w)$,
the length of a reduced word for w .