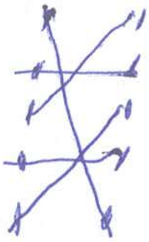
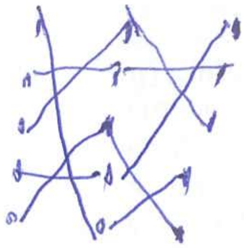
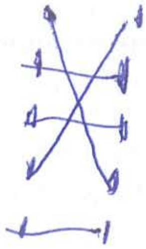
The generators and relations for S_n

Generators A: Permutation matrices

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} =$$ 

Relations A: $vw = u$ is given by matrix multiplication

 $=$ 

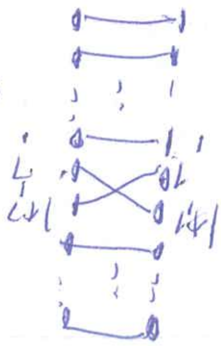
$v \quad w \qquad\qquad u$

Generators B $s_1, \ldots, s_{n-1}$

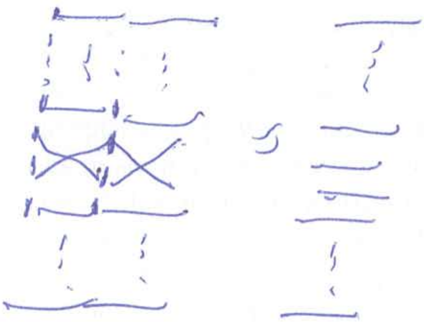
Relations B $s_j^2 = 1$, $s_j s_k = s_k s_j$ if $k \notin \{j-1, j+1\}$

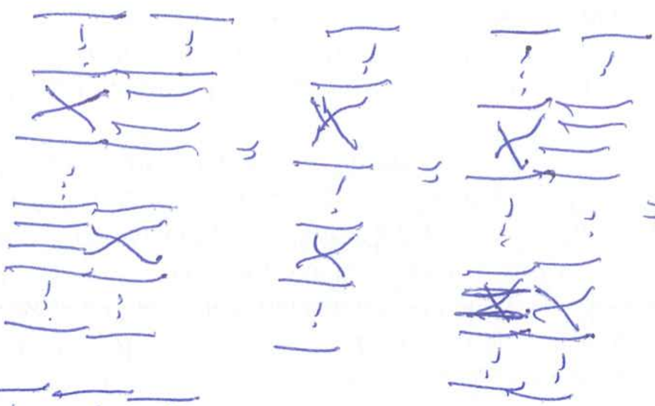
and $s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1}$.

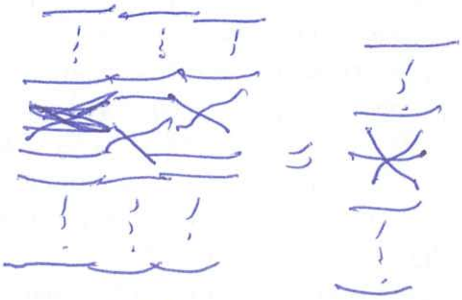
Generators B in terms of Generators A

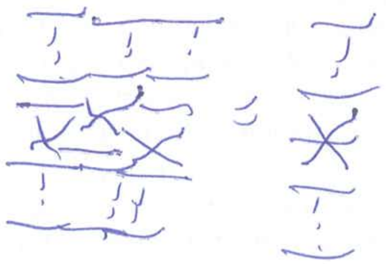
$s_i =$  for $i \in \{1, \ldots, n-1\}$

Relations B from relations A

$s_i^2 =$  $= 1$.

$s_j s_k =$  $= s_k s_j$

$s_i s_{i+1} s_i =$  and

$s_{i+1} s_i s_{i+1} =$ . So $s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1}$.

Generators A from Generators B (the subject of reduced words)

Let $w \in S_n$.

A reduced word for w is an expression

$$w = s_{i_1} \cdots s_{i_\ell} \quad \text{with } i_1, \dots, i_\ell \in \{1, \dots, n\}$$

and ℓ minimal. The length of w is $\ell(w)$, the length of a reduced word for w.