1 Single pages

1.1 Numbers and intervals

The positive integers: $\mathbb{Z}_{>0} = \{1, 2, 3, \ldots\}$.

The nonnegative integers: $\mathbb{Z}_{\geq 0} = \{0, 1, 2, 3, \ldots\}$.

The integers: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

The rational numbers: $\mathbb{Q} = \left\{ \frac{a}{b} \mid a \in \mathbb{Z},\, b \in \mathbb{Z}_{\neq 0} \text{ and } \frac{a}{b} = \frac{c}{d} \text{ if } ad = bc \right\}$.

The real numbers:

$$\mathbb{R} = \{\pm a_\ell a_{\ell-1} \ldots a_1 a_0 . a_{-1} a_{-2} \ldots \mid \ell \in \mathbb{Z}_{\geq 0},\, a_i \in \{0, \ldots, 9\},\, a_\ell \neq 0 \text{ if } \ell > 0\}.$$

with a convention that if $a_k \neq 9$ then $\pm a_\ell ... a_{k+1} a_k 9999... = \pm a_\ell ... a_{k+1}(a_k + 1)000...$
so that, for example, $0.9999... = 1.0000....$

The complex numbers:

$$\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R}\} \qquad \text{with } i^2 = -1.$$

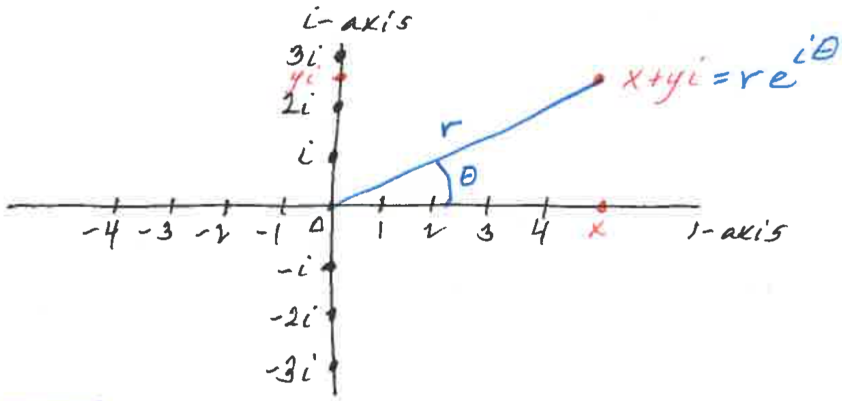
Let $a, b \in \mathbb{R}$ with $a < b$. Define

$$\mathbb{R}_{(a,b)} = \{x \in \mathbb{R} \mid a < x < b\}, \qquad \mathbb{R}_{[a,b)} = \{x \in \mathbb{R} \mid a \leq x < b\}$$

$$\mathbb{R}_{(a,b]} = \{x \in \mathbb{R} \mid a < x \leq b\}, \qquad \mathbb{R}_{[a,b]} = \{x \in \mathbb{R} \mid a \leq x \leq b\}$$

$$\mathbb{R}_{(a,\infty)} = \{x \in \mathbb{R} \mid a < x\}, \qquad \mathbb{R}_{[a,\infty)} = \{x \in \mathbb{R} \mid a \leq x\}$$

$$\mathbb{R}_{(\infty,a)} = \{x \in \mathbb{R} \mid x < a\}, \qquad \mathbb{R}_{(\infty,a]} = \{x \in \mathbb{R} \mid x \leq a\}.$$



Picture of $\mathbb{Z} \subseteq \mathbb{R} \subseteq \mathbb{C}$

Remark 1.1.

$\frac{1}{a}$ is the number that when multiplied by a gives 1.

□