

**13.12 Series**

1. Prove that  $\sum_{n=1}^r n = \frac{1}{2}r(r+1)$ .
2. Prove that  $\sum_{n=1}^r \frac{1}{n(n+1)} = \frac{r}{r+1}$ .
3. Let  $a \in \mathbb{R}$  with  $a \neq 1$ . Prove that  $\sum_{n=0}^r a^n = \frac{1-a^{r+1}}{1-a}$ .
4. Determine if the series  $\sum_{n=1}^{\infty} \frac{1}{n^5}$  converges in  $\mathbb{R}$ .
5. Determine if the series  $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$  converges in  $\mathbb{R}$ .
6. Determine if the series  $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$  converges in  $\mathbb{R}$ .
7. Determine if the series  $\sum_{n=2}^{\infty} \frac{1}{(n-1)^2}$  converges in  $\mathbb{R}$ .
8. Determine if the series  $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$  converges in  $\mathbb{R}$ .
9. Determine if the series  $\sum_{n=2}^{\infty} \frac{n}{n^3-1}$  converges in  $\mathbb{R}$ .
10. Determine if the series  $\sum_{n=1}^{\infty} \frac{1}{n+1}$  converges in  $\mathbb{R}$ .
11. Determine if the series  $\sum_{n=2}^{\infty} \frac{1}{n-1}$  converges in  $\mathbb{R}$ .
12. Determine if the series  $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$  converges in  $\mathbb{R}$ .
13. Determine if the series  $\sum_{n=2}^{\infty} \frac{n}{\sqrt{n}-1}$  converges in  $\mathbb{R}$ .
14. Determine if the series  $\sum_{n=1}^{\infty} \frac{2}{3^n+1}$  converges in  $\mathbb{R}$ .
15. Determine if the series  $\sum_{n=1}^{\infty} \frac{3^n+1}{4^n+1}$  converges in  $\mathbb{R}$ .

16. Determine if the series  $\sum_{n=1}^{\infty} \frac{n^3}{2^n}$  converges in  $\mathbb{R}$ .
17. Determine if the series  $\sum_{n=1}^{\infty} \frac{n!}{n^n}$  converges in  $\mathbb{R}$ .
18. Determine if the series  $\sum_{n=1}^{\infty} \frac{2^n}{n+1}$  converges in  $\mathbb{R}$ .
19. Determine if the series  $\sum_{n=1}^{\infty} \frac{2^n}{n!}$  converges in  $\mathbb{R}$ .
20. Determine if the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/3}}$  converges in  $\mathbb{R}$ .
21. Determine if the series  $\sum_{n=1}^{\infty} \sqrt{\frac{n}{n+1}}$  converges in  $\mathbb{R}$ .
22. Determine if the series  $\sum_{n=1}^{\infty} \frac{1}{n^7}$  converges in  $\mathbb{R}$ .
23. Determine if the series  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+n}}$  converges in  $\mathbb{R}$ .
24. Determine if the series  $\sum_{n=1}^{\infty} \frac{n^3}{4^n}$  converges in  $\mathbb{R}$ .
25. Determine if the series  $\sum_{n=1}^{\infty} \frac{\sin(n)}{1+n^2}$  converges in  $\mathbb{R}$ .
26. Determine if the series  $\frac{2}{1} - \frac{2}{2} + \frac{2}{3} - \frac{2}{4} + \frac{2}{5} - \dots$  converges in  $\mathbb{R}$ .
27. Determine if the series  $-\frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \frac{4}{5} - \frac{5}{6} + \dots$  converges in  $\mathbb{R}$ .
28. Determine if the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\log(n+1)}$  converges in  $\mathbb{R}$ .
29. Determine if the series  $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$  converges in  $\mathbb{R}$ .
30. Determine if the series  $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$  converges in  $\mathbb{R}$ .
31. Determine  $\sum_{n=1}^{\infty} \left(\frac{1}{10}\right)^{n-1}$ .
32. Determine  $\sum_{n=1}^{\infty} n\left(\frac{1}{5}\right)^{n-1}$ .

33. Determine  $\sum_{n=0}^{\infty} \frac{\left(\frac{1}{7}\right)^{n+1}}{n+1}$ .

34. Determine  $\sum_{n=1}^{\infty} \frac{\left(\frac{1}{3}\right)^n}{n}$ .

35. Determine  $\sum_{n=1}^{\infty} \frac{n}{3^{n-1}}$ .

36. Determine  $\sum_{n=1}^{\infty} \frac{1}{n2^{n+1}}$ .

37. Determine  $\sum_{n=1}^{\infty} n(n-1)\left(\frac{1}{4}\right)^n$ .