1.3   Tutorial 1 MAST30005 Semester 1, 2024: Proof machine

1. Why should the symbols $\subset$, $\forall$, $\exists$ be banned? What should they be replaced by?

2. Why should the phrase "Let $a > 7$" be banned? What should it be replaced by?

3. What does the symbol $\mapsto$ mean and how should it be used?

4. What comes at the end of a sentence? and at the end of an equation that ends a sentence?

5. Why should the phrases 'for all', 'for every', 'for each', and 'for some' be banned? What should they be replaced by?

6. Why is it bad style to start a sentence with a mathematical symbol? What should be written instead?

7. Why do we never use a comma in place of the word 'then' in mathematical writing?

8. What are the symbols for "subset of", "proper subset of"? "element of" and "equal"?

9. What is the form of a mathematical definition (for a noun)?

10. What is the form of a mathematical definition (for an adjective)?

11. What is the definition of equal sets?

12. What is the defnition of equal functions?

13. What is the definition of a function?

14. What is Proof type I? How does proof type I proceed?

15. What is proof type II? How does proof type II proceed?

16. What is proof type III? How does proof type III proceed?

17. What is proof by contrapositive? How does a proof by contrapositive proceed?

18. How do proofs of uniqueness proceed?

19. What is the underlying source of proof by induction? How does proof by induction proceed?

20. Why should proof by contradiction be banned? What should it be replaced by?

21. What is the structure of a universal property? What property 'in English' is a universal property capturing? Give an explicit example of something that is defined by a universal property and state the definition carefully and completely.

22. What property is "there exists" capturing? What property is "there exists a unique" capturing?

23. Prove that if $x^2 < y^2$ then $x < y$. (Correct the statement as necessary before proving it.)

24. Prove that if a^2 is divisible by 2 then a^2 is divisible by 4. (Correct the statement as necessary before proving it.)

25. Prove that a function is invertible if and only if it is bijective. (Correct the statement as necessary before proving it.)

26. When is it appropriate to use the symbols $\implies$, $\iff$, $\longrightarrow$ and when is it not? When they should not be used, what should they be replaced by?