Confusing symbols:

(1) $\subset$ could mean $\subseteq$ or $\subsetneq$

(2) $a>7$ could mean $a\in\mathbb{Z}_{>7}$ or $a\in\mathbb{Q}_{>7}$ or $a\in\mathbb{R}_{>7}$

(3) If $a\in A, b\in B$ could mean if $a\in A$ and $b\in B$
or if $a\in A$ then $b\in B$

Use comma only for "and" in a mathematical context

(4) Symbols that alienate our society from mathematics and whose use increases the fears and insecurities many people harbor in regard to mathematics

(1) $\forall$ can be replaced by "if"

(2) $\exists$ can be replaced by "there exists"

(3) $\Rightarrow$ can be replaced by "so"

(4) $\Leftrightarrow$ can be replaced by "if and only if."

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The sources of logical constructs

(1) "There exist $s \in S$ such $s < t$"

captures the statement

$$\{s \in S \mid s < t\} \neq \emptyset \quad \text{or} \quad \mathrm{Card}(\{s \in S \mid s < t\}) = 0.$$

(2) "There exists a unique $s \in S$ such that $s < t$"

captures the statement

$$\mathrm{Card}(\{s \in S \mid s < t\}) = 1.$$

(3) The proof by induction mechanics:

Base case: $n = 1$ . . . .

Base case: $n = 2$ . . . .

Base case: $n = 3$, ...

Induction step: . . . .

captures the definition of $\mathbb{Z}_{\geq 0}$.

(4) "x is finite"

captures the statement

$x \in \mathbb{Z}_{\geq 0}$.

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Addressing the hidden question / Addressing the point

(1) "Show that $3\cdot 6 = 1\cdot 6$ in $\mathbb{Z}/12\mathbb{Z}$" is noting that $\mathbb{Z}/12\mathbb{Z}$ does not satisfy the cancellation law and is not an integral domain. The statement $3\cdot 6 = 1\cdot 6$ is equivalent to $3\cdot 6 - 1\cdot 6 = 0$ and equivalent to $(3-1)\cdot 6 = 0$. Show that $2\cdot 6 = 0$ gives that $\mathbb{Z}/12\mathbb{Z}$ has the nonzero elements 2 and 6 as zero divisors, which is an alternate way of noting that $\mathbb{Z}/12\mathbb{Z}$ is not an integral domain.

Addressing the incorrectly or imprecisely stated question

(1) Give an example of $\varphi: G \to H$, a group homomorphism. Show that $\mathrm{im}(\varphi)$ is not a subgroup of H

The right question is: Is $\mathrm{im}(\varphi)$ a subgroup of G?

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Let $\varphi: G \to H$ be a group homomorphism

To show: (a) $1 \in \operatorname{im}\varphi$

(b) If $h_1, h_2 \in H$ then $h_1 h_2 \in \operatorname{im}\varphi$

(c) If $h \in H$ then $h^{-1} \in \operatorname{im}\varphi$.

(a) Since $\varphi(1_G) = 1_H$, where 1_G is the identity in G and 1_H is the identity in H then $1_H \in \operatorname{im}(\varphi)$

(b) Assume $h_1, h_2 \in \operatorname{im}(\varphi)$.

Then there exist $g_1, g_2 \in G$ such that $\varphi(g_1) = h_1$ and $\varphi(g_2) = h_2$.

Then $\varphi(g_1 g_2) = \varphi(g_1)\varphi(g_2) = h_1 h_2$

So $h_1 h_2 \in \operatorname{im}(\varphi)$

(c) Assume $h \in \operatorname{im}(\varphi)$.

Then there exists $g \in G$ such that $\varphi(g) = h$.

Then $\varphi(g^{-1})\varphi(g) = \varphi(g^{-1}g) = \varphi(1_G) = 1_H$.

and $\varphi(g)\varphi(g^{-1}) = \varphi(g g^{-1}) = \varphi(1_G) = 1_H$.

So $\varphi(g^{-1}) = \varphi(g)^{-1} = h^{-1}$ and so $h^{-1} \in \operatorname{im}(\varphi)$.

Let $n \in \mathbb{Z}_{>0}$

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An $n\times n$ <u>permutation matrix</u> is an $n\times n$ matrix

with (a) exactly one nonzero entry in each row and column and

(b) the nonzero entries are 1.

Let $S_n = \{n\times n \text{ permutation matrices}\}$

Let $c \in S_n$ be the matrix given by

$$c_{i,i+1} = 1 \quad \text{for } i \in \{1, \ldots, n-1\},$$

and $c_{n,1} = 1$.

Let $C_n = \{1, c, c^2, \ldots, c^{n-1}\}$. The <u>cyclic matrices</u> are the elements of C_n

Let $w_0 \in S_n$ be the matrix given by

$$(w_0)_{i,n-i+1} = 1 \quad \text{for } i \in \{1, \ldots, n\}.$$

Let

$$D_n = \{1, c, c^2, \ldots, c^{n-1}, w_0, cw_0, \ldots, c^{n-1}w_0\}.$$

The <u>dihedral matrices</u> are the elements of D_n.