Let V be an $\mathbb{F}$-vector space

A subspace of V is a subset $W \subseteq V$ such that

(a) If $w_1, w_2 \in W$ then $w_1 + w_2 \in W$.

(b) If $w \in W$ and $c \in \mathbb{F}$ then $cw \in W$.

(c) $0 \in W$.

Let $k \in \mathbb{Z}_{>0}$.

Let $S = \{v_1, v_2, \ldots, v_k\}$ be a subset of V with k elements

The subspace generated by S is

$$\text{span}(S) = \{c_1 v_1 + \cdots + c_k v_k \mid c_1, \ldots, c_k \in \mathbb{F}\}$$

A linear combination of $v_1, \ldots, v_k$ is an element of $\text{span}(S)$.

The set S is linearly independent if S satisfies the condition

if $c_1, \ldots, c_k \in \mathbb{F}$ and $c_1 v_1 + \cdots + c_k v_k = 0$

then $c_1 = 0$ and $c_2 = 0$ and $\ldots$ and $c_k = 0$.

A basis of V is a subset $B \subseteq V$ such that

(a) B is linearly independent

(b) $\text{span}(B) = V$.

Topic 4 Example 7 Show that

$S = \{(x, y, z) \in \mathbb{R}^3 \mid x+y+z=0\}$ is a subspace of $\mathbb{R}^3$

Topic 4 Example 8 Show that

$L = \{(x, y) \in \mathbb{R}^2 \mid y = 2x+1\}$ is a subspace of $\mathbb{R}^2$.

Topic 4 Example 9 Show that

$S = \{a_1 x + a_2 x^2 \mid a_1, a_2 \in \mathbb{R}\}$ is a subspace of P_2

Topic 4 Example 10 Show that

$W = \{A \in M_{2\times 2}(\mathbb{R}) \mid \text{Tr}(A) = 0\}$ is a subspace of $M_{2\times 2}(\mathbb{R})$.

Topic 4 Example 11 Show that

$W = \{A \in M_{2\times 2}(\mathbb{R}) \mid \det(A) = 0\}$ is a subspace of $M_{2\times 2}(\mathbb{R})$.

Topic 4 Example 13 Is

$\langle 1, 2, 3 \rangle \in \text{span}\{\langle 1, -1, 2 \rangle, \langle -1, 1, 2 \rangle\}$?

Topic 4 Example 14 Is

$1 - 2x - x^2 \in \text{span}\{1 + x + x^2, 3 + x^2\}$?

Topic 4 Example 15 What is

$\text{span}\{\langle 1, 1, 1 \rangle, \langle 2, 2, 2 \rangle, \langle 3, 3, 3 \rangle\}$?

Topic 4 Example 16 Show that

$\text{span}\{\langle 1, -1 \rangle, \langle 2, 4 \rangle\} = \mathbb{R}^2$.

Topic 4 Example 17 Show that

$\text{span}\{\langle 1, 2, 0 \rangle, \langle 1, 5, 3 \rangle, \langle 0, 1, 1 \rangle\} = \mathbb{R}^3$

Topic 4 Example 18 Show that

$\text{span}\{1 + x + x^2, x^2\} = P_2$.

Topic 4 Example 19

(a) Is $S=\{|2,-1,1\rangle, |-6,3,-3\rangle\}$ linearly independent?

(b) Is $S=\{|2,-1,1\rangle, |4,0,2\rangle\}$ linearly independent?

Topic 4 Example 20

Is $S=\{|2,0,0\rangle, |4,1,7\rangle, |2,-1,2\rangle\}$ linearly independent?

Topic 4 Example 21

Is $S=\{2+2x+5x^2, 1+x+x^2, 1+2x+3x^2\}$ linearly independent?

Topic 4 Example 22

Is $S=\left\{\begin{pmatrix}1&3\\1&1\end{pmatrix}, \begin{pmatrix}-2&1\\1&-1\end{pmatrix}, \begin{pmatrix}1&10\\4&2\end{pmatrix}\right\}$ linearly independent?