Topic 2. Example 8. The system of equations

$$\begin{aligned} x+2y+z&=-4,\\ -x-y+z&=11,\\ y+3z&=21, \end{aligned} \qquad \text{is} \qquad \begin{pmatrix} 1 & 2 & 1\\ -1 & -1 & 1\\ 0 & 1 & 3 \end{pmatrix}\begin{pmatrix} x\\ y\\ z\end{pmatrix} = \begin{pmatrix} -4\\ 11\\ 21\end{pmatrix}$$

and multiplying both sides by

$$\begin{pmatrix} 1 & 2 & 1\\ -1 & -1 & 1\\ 0 & 1 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} -4 & -5 & 3\\ 3 & 3 & -2\\ -1 & -1 & 1 \end{pmatrix} \qquad \text{(see Topic 2 Example 5)}$$

gives

$$\begin{pmatrix} x\\ y\\ z\end{pmatrix} = \begin{pmatrix} 1 & 2 & 1\\ -1 & -1 & 1\\ 0 & 1 & 3 \end{pmatrix}^{-1}\begin{pmatrix} -4\\ 11\\ 21\end{pmatrix} = \begin{pmatrix} -4 & -5 & 3\\ 3 & 3 & -2\\ -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} -4\\ 11\\ 21\end{pmatrix} = \begin{pmatrix} 16-55+63\\ -12+33-42\\ 4-11+21\end{pmatrix} = \begin{pmatrix} 24\\ -21\\ 14\end{pmatrix}.$$

Topic 2. Example 7.

$$A=\begin{pmatrix} 1 & 2\\ 3 & 4\end{pmatrix} = y_1(\tfrac{1}{3})\begin{pmatrix} 3 & 4\\ 0 & \frac{2}{3}\end{pmatrix} = y_1(\tfrac{1}{3})h(3,\tfrac{2}{3})\begin{pmatrix} 1 & \frac{4}{3}\\ 0 & 1\end{pmatrix} = y_1(\tfrac{1}{3})h(3,\tfrac{2}{3})x_{12}(\tfrac{4}{3})\cdot 1_2.$$

Then $\det(A)=(-1)\cdot 3\cdot \frac{2}{3}\cdot 1=-2$ and $\mathrm{rank}(A)=2$ and

$$\begin{aligned} A^{-1} &= x_{12}(\tfrac{4}{3})^{-1}h(3,\tfrac{2}{3})^{-1}y_1(\tfrac{1}{3})^{-1} = \begin{pmatrix} 1 & -\frac{4}{3}\\ 0 & 1\end{pmatrix}\begin{pmatrix} \frac{1}{3} & 0\\ 0 & \frac{3}{2}\end{pmatrix}\begin{pmatrix} 0 & 1\\ 1 & -\frac{1}{3}\end{pmatrix} = \begin{pmatrix} 1 & -\frac{4}{3}\\ 0 & 1\end{pmatrix}\begin{pmatrix} 0 & \frac{1}{3}\\ \frac{3}{2} & -\frac{1}{2}\end{pmatrix}\\ &= \begin{pmatrix} -2 & 1\\ \frac{3}{2} & -\frac{1}{2}\end{pmatrix}. \end{aligned}$$

Topic 2. Example 9. The matrix

$$\begin{aligned} A &= \begin{pmatrix} 1 & -1 & 2 & 1\\ 0 & 1 & 1 & -2\\ 1 & -3 & 0 & 5\end{pmatrix}\\ &= y_2(0)\begin{pmatrix} 1 & -1 & 2 & 1\\ 1 & -3 & 0 & 5\\ 0 & 1 & 1 & -2\end{pmatrix}\\ &= y_2(0)y_1(1)\begin{pmatrix} 1 & -3 & 0 & 5\\ 0 & 2 & 2 & -4\\ 0 & 1 & 1 & -2\end{pmatrix}\\ &= y_2(0)y_1(1)y_2(2)\begin{pmatrix} 1 & -3 & 0 & 5\\ 0 & 1 & 1 & -2\\ 0 & 0 & 0 & 0\end{pmatrix}\\ &= y_2(0)y_1(1)y_2(2)\cdot 1_2\cdot x_{12}(-3)x_{13}(5)x_{24}(-2) \end{aligned}$$

has rank 2. Since A is not square then $\det(A)$ and A^{-1} are not defined.