Topic 2. Example 5. Let

$$A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & 1 \\ 0 & 1 & 3 \end{pmatrix}$$

Since

$$\begin{aligned}
&x_{23}(3)^{-1}x_{13}(-4)^{-1}x_{12}(1)^{-1}d(-1,1,-1)^{-1}y_2(1)^{-1}y_1(-1)^{-1}A \\
&\quad = x_{23}(3)^{-1}x_{13}(-4)^{-1}x_{12}(1)^{-1}d(-1,1,-1)^{-1}y_2(1)^{-1} \begin{pmatrix} -1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 3 \end{pmatrix} \\
&\quad = x_{23}(3)^{-1}x_{13}(-4)^{-1}x_{12}(1)^{-1}d(-1,1,-1)^{-1} \begin{pmatrix} -1 & -1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & -1 \end{pmatrix} \\
&\quad = x_{23}(3)^{-1}x_{13}(-4)^{-1}x_{12}(1)^{-1} \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \\
&\quad = x_{23}(3)^{-1}x_{13}(-4)^{-1} \begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \\
&\quad = x_{23}(3)^{-1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

then

$$\begin{aligned}
A^{-1} &= x_{23}(3)^{-1}x_{13}(-4)^{-1}x_{12}(1)^{-1}d(-1,1,-1)^{-1}y_2(1)^{-1}y_1(-1)^{-1} \\
&= x_{23}(3)^{-1}x_{13}(-4)^{-1}x_{12}(1)^{-1}d(-1,1,-1)^{-1}y_1(1)^{-1} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= x_{23}(3)^{-1}x_{13}(-4)^{-1}x_{12}(1)^{-1}d(-1,1,-1)^{-1} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & -1 \end{pmatrix} \\
&= x_{23}(3)^{-1}x_{13}(-4)^{-1}x_{12}(1)^{-1} \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 1 \end{pmatrix} \\
&= x_{23}(3)^{-1}x_{13}(-4)^{-1} \begin{pmatrix} 0 & -1 & -1 \\ 0 & 0 & 1 \\ -1 & -1 & 1 \end{pmatrix} \\
&= x_{23}(3)^{-1} \begin{pmatrix} -4 & -5 & 3 \\ 0 & 0 & 1 \\ -1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} -4 & -5 & 3 \\ 3 & 3 & -2 \\ -1 & -1 & 1 \end{pmatrix}
\end{aligned}$$

and

$$A^{-1} = \begin{pmatrix} -4 & -5 & 3 \\ 3 & 3 & -2 \\ -1 & -1 & 1 \end{pmatrix} = \frac{1}{(-3-1)+(6-1)} \begin{pmatrix} -3-1 & (-1)(-3-0) & -1-0 \\ (-1)(6-1) & 3-0 & (-1)(1-0) \\ 2+1 & (-1)(1+1) & -1+2 \end{pmatrix}^t$$