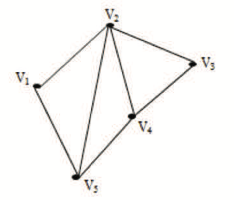
Topic 2. Examples 1 and 2. The graph



has adjacency matrix $A = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$

Then

$$A^3 = A(A^2) = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 & 2 & 1 \\ 1 & 4 & 1 & 2 & 2 \\ 1 & 1 & 2 & 1 & 2 \\ 2 & 2 & 1 & 3 & 1 \\ 1 & 2 & 2 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 6 & 3 & 3 & 5 \\ 6 & 6 & 6 & 7 & 7 \\ 3 & 6 & 2 & 5 & 3 \\ 3 & 7 & 5 & 4 & 7 \\ 5 & 7 & 3 & 7 & 4 \end{pmatrix}.$$

Topic 2. Example 3.

$$\text{Let} \qquad A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}. \qquad \text{Then} \qquad A^t = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}.$$

Topic 2. Example 4.

$$\text{If} \qquad A = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} \qquad \text{then} \qquad A^{-1} = \tfrac{1}{3} \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$$

since $\frac{1}{3} \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = 1$ and

$$\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} \tfrac{1}{3} \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1.$$

The normal form for A is

$$A = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} = y_1(2) \begin{pmatrix} 1 & 1 \\ 0 & -3 \end{pmatrix} = y_1(2) d(1, -3) \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = y_1(2) d(1, -3) x_{12}(1) \cdot 1_2$$

and $\det(A) = (-1) \cdot 1 \cdot 3 \cdot 1 = -3$ and $\mathrm{rank}(A) = 2$.