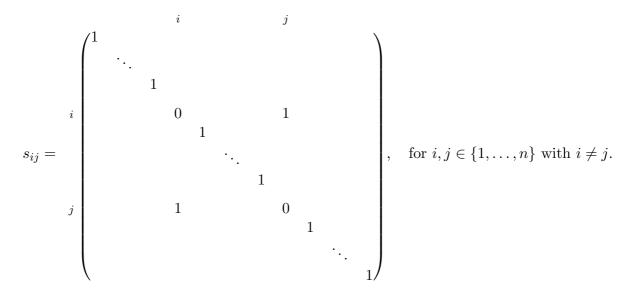
4.2 Permutation matrices S_n

Let $n \in \mathbb{Z}_{>0}$.

- An $n \times n$ permutation matrix is an $n \times n$ matrix such that
 - (a) There is exactly one nonzero entry in each row and each column,
 - (b) The nonzero entries are 1.
- The symmetric group is

 $S_n = \{n \times n \text{ permutation matrices}\}.$

• The transpositions in S_n are the matrices $s_{ij} = 1 - E_{ii} - E_{jj} + E_{ij} + E_{ji}$,



• The simple transpositions in S_n are the matrices $s_i = s_{i,i+1}$,

Proposition 4.2. The symmetric group S_n is presented by generators $s_1, s_2, \ldots, s_{n-1}$ and relations

$$s_{i}^{2} = 1 \quad and \quad s_{j}s_{j+1}s_{j} = s_{j+1}s_{j}s_{j+1} \quad and \quad s_{k}s_{\ell} = s_{\ell}s_{k},$$
(4.2)
for $i, j, k, \ell \in \{1, \dots, n-1\}$ with $j \neq n-1$ and $k \neq \ell \pm 1$.

A key step of the proof is to write a permutation matrix in normal form.