Calculus 2 Lect. 34

21.10.2019
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Example 7.22 $\int (3x^2y + 12y^2x^3)\,dx$

Solution $\int (3x^2y + 12y^2x^3)\,dx = x^3y + 3y^2x^4 + c$,

where c is a "constant," which could depend on y, because

$$\frac{\partial}{\partial x}(x^3y + 3y^2x^4 + c) = 3x^2y + 3y^2 4x^3 + 0$$

$$= 3x^2y + 12y^2x^3. \quad //$$

Example 7.23 Evaluate $\int_0^1 (3x^2y + 12y^2x^3)\,dy$.

Solution: $\int_0^1 (3x^2y + 12y^2x^3)\,dy$

$$= \int_{y=0}^{y=1} (3x^2y + 12y^2x^3)\,dy$$

$$= \left[3x^2\frac{y^2}{2} + 12\frac{y^3}{3}x^3 + c \right]_{y=0}^{y=1}$$

$$= \left(3x^2\frac{1}{2} + 12\cdot\frac{1}{3}x^3 + c\right) - \left(3x^2\cdot\frac{0^2}{2} + 12\cdot\frac{0^3}{3}x^3 + c\right)$$

$$= \frac{3}{2}x^2 + 4x^3 \quad //$$

Example 7.24 Evaluate $\iint_R (x^2+y^2)\,dx\,dy$ with $R=[-1,1]\times[0,1]$.

Solution:

$$\iint_R (x^2+y^2)\,dx = \int_{y=0}^{y=1}\int_{x=-1}^{x=1} (x^2+y^2)\,dx\,dy$$

$$= \int_{y=0}^{y=1} \left(\tfrac{1}{3}x^3 + y^2 x \Big]_{x=-1}^{x=1} \right) dy$$

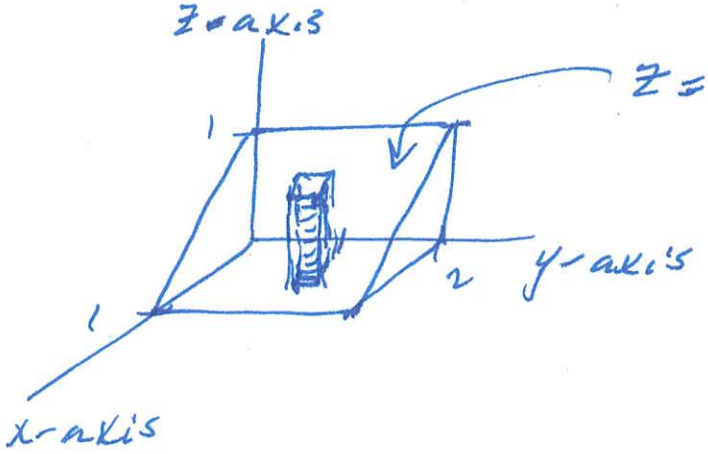
$$= \int_{y=0}^{y=1} \left(\left(\tfrac{1}{3}\cdot 1^3 + y^2\cdot 1\right) - \left(\tfrac{1}{3}(-1)^3 + y^2(-1)\right) \right) dy$$

$$= \int_{y=0}^{y=1} \left(\tfrac{1}{3} + y^2 + \tfrac{1}{3} + y^2\right) dy = \int_{y=0}^{y=1} \left(\tfrac{2}{3} + 2y^2\right) dy$$

$$= \tfrac{2}{3}y + 2\cdot\tfrac{1}{3}y^3 \Big]_{y=0}^{y=1} = \left(\tfrac{2}{3} + \tfrac{2}{3}\right) - (0+0)$$

$$= \frac{4}{3}.$$

Example 7.25 Find the volume of



Solution $\int_{y=0}^{y=2}\int_{x=0}^{x=1} (z_{top} - z_{bottom})\, dx\, dy$

$= \int_{y=0}^{y=2}\int_{x=0}^{x=1} ((1-x)-0)\, dx\, dy = \int_{y=0}^{y=2}\int_{x=0}^{x=1} (1-x)\, dx\, dy$

$= \int_{y=0}^{y=2} \left(x - \frac{1}{2}x^2\right)\Big]_{x=0}^{x=1} dy = \int_{y=0}^{y=2} \left(\left(1-\frac{1}{2}\right) - (0-0)\right) dy$

$= \int_{y=0}^{y=2} \frac{1}{2}\, dy = \frac{1}{2}y\Big]_{y=0}^{y=2} = \frac{1}{2}\cdot 2 - \frac{1}{2}\cdot 0 = 1 - 0 = 1.$ //