Calculus 2 Lecture 28 07.10.2019

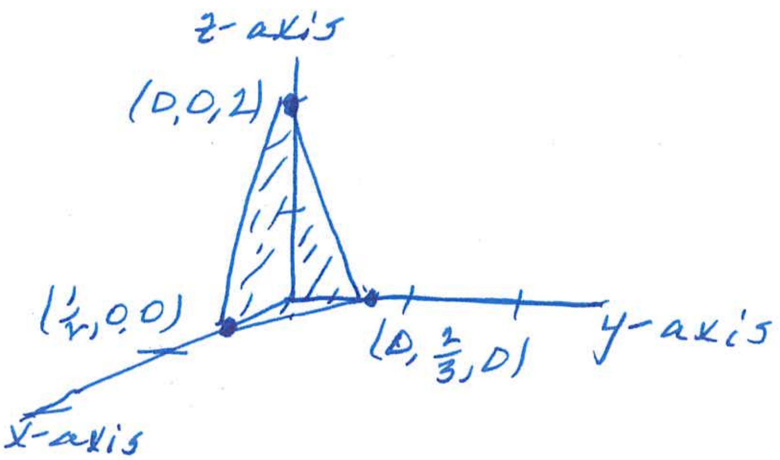
A. Rem

Example 7.1 Let $f(x,y) = 2-4x-3y$. Graph

$$\{(x,y,z)\in\mathbb{R}^3 \mid z=f(x,y)\} = \{(x,y,z)\in\mathbb{R}^3 \mid 4x+3y+z=2\}.$$

Solution: This the graph of a plane which contains the points

$(0,0,2)$ and $(0,\frac{2}{3},0)$ and $(\frac{2}{4},0,0)$



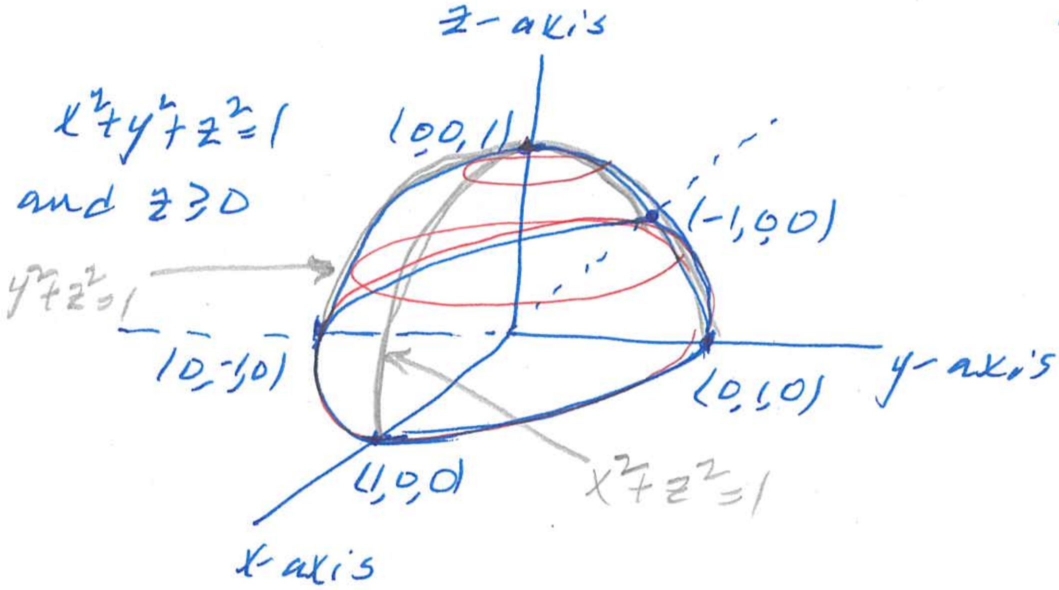
Example 7.2 Graph $z=\sqrt{1-x^2-y^2}$ and level curves.

Solution $\{(x,y,z)\in\mathbb{R}^3 \mid z=\sqrt{1-x^2-y^2}\}$

$= \{(x,y,z)\in\mathbb{R}^3 \mid x^2+y^2+z^2=1 \text{ and } z\geq 0\}$

$= \{(x,y,z)\in\mathbb{R}^3 \mid (x,y,z) \text{ is distance 1 from } (0,0,0) \text{ and } z\geq 0\}$

This is the $z\geq 0$ of a sphere of radius 1.

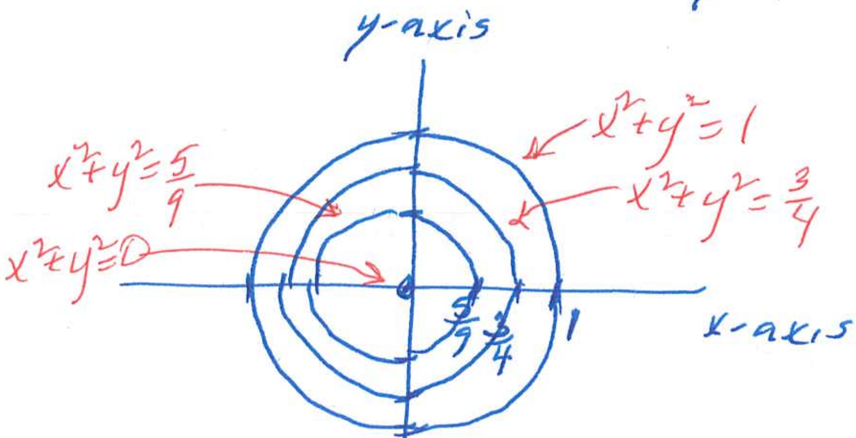


Level curves are $0=\sqrt{1-x^2-y^2}$ i.e. $x^2+y^2=1$

$\frac{1}{2}=\sqrt{1-x^2-y^2}$ i.e. $x^2+y^2=\frac{3}{4}$

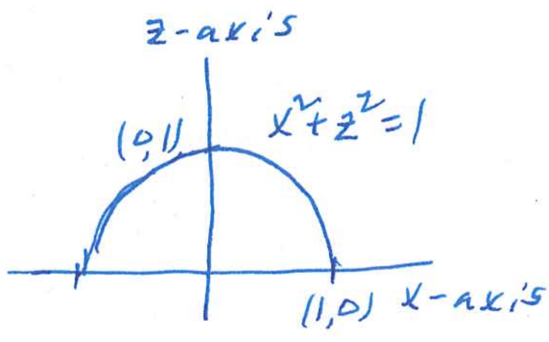
$\frac{2}{3}=\sqrt{1-x^2-y^2}$, i.e. $x^2+y^2=\frac{5}{9}$

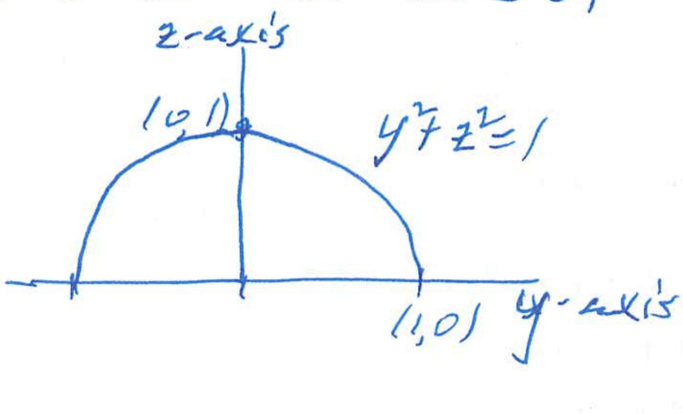
$1=\sqrt{1-x^2-y^2}$, i.e. $x^2+y^2=0$



Cross sections are $z=\sqrt{1-0^2-y^2}$ i.e. $y^2+z^2=1$

$z=\sqrt{1-x^2-0^2}$ i.e. $x^2+z^2=1$





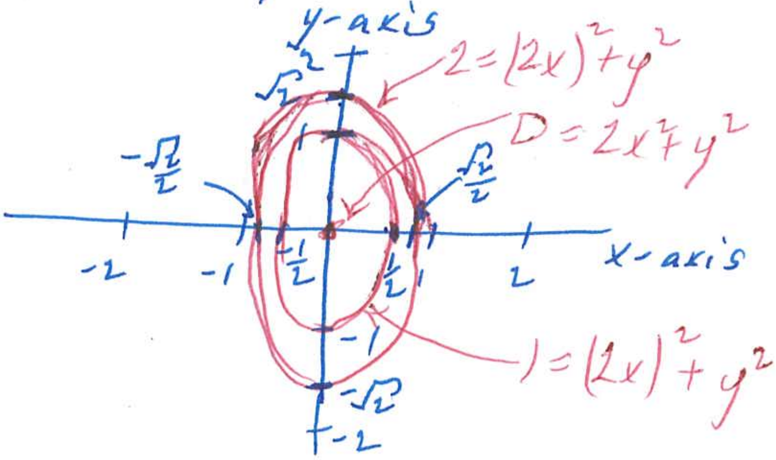
Example 7.3 Sketch the graph of $z = 4x^2 + y^2$

Solution Level curves are $0 = 4x^2 + y^2 = (2x)^2 + y^2$,

$1 = (2x)^2 + y^2$, and

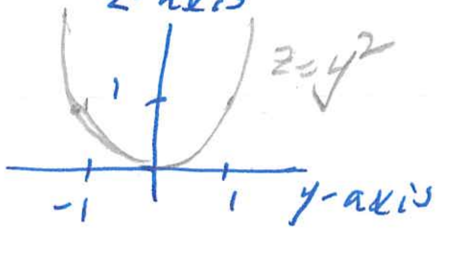
$2 = (2x)^2 + y^2$.

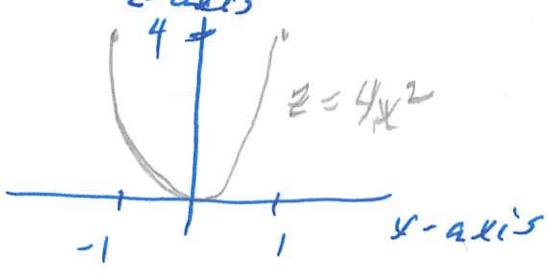
These are ellipses:



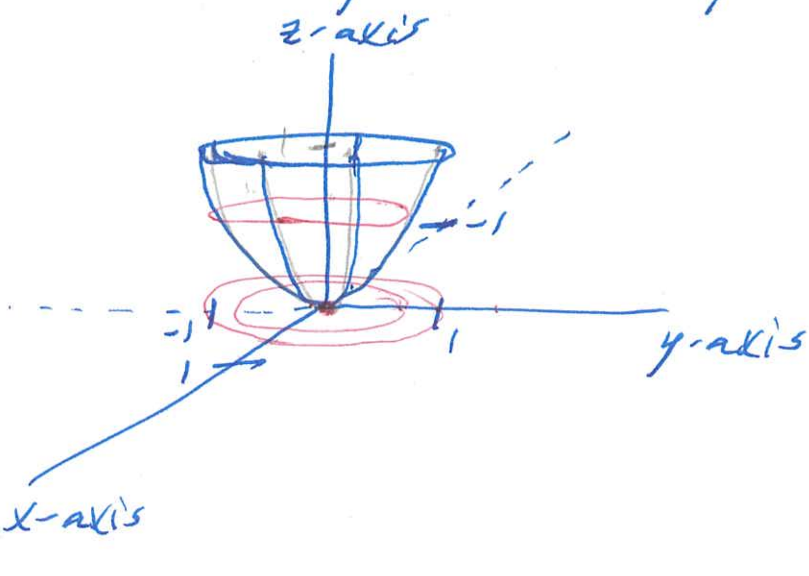
Cross sections are $z = 4 \cdot 0^2 + y^2 = y^2$, i.e. $z = y^2$

and $z = 4x^2 + 0^2 = 4x^2$, i.e. $z = 4x^2$





The graph of $z = 4x^2 + y^2$ is an elliptic paraboloid.



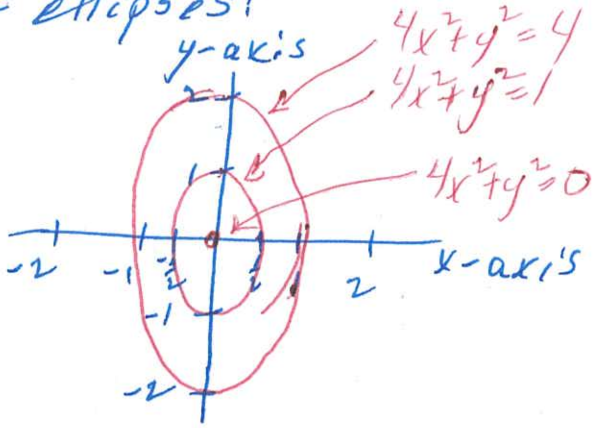
Example 7.4 Sketch the graph of $z=\sqrt{4x^2+y^2}$.

Solution Level curves are $0=\sqrt{4x^2+y^2}$, i.e. $4x^2+y^2=0$

$1=\sqrt{4x^2+y^2}$, i.e. $4x^2+y^2=1$

and $2=\sqrt{4x^2+y^2}$, i.e. $4x^2+y^2=4$

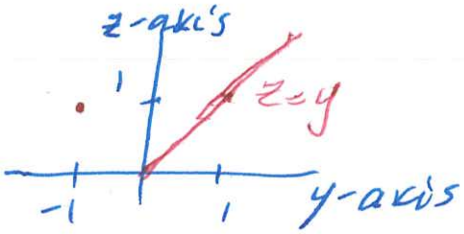
These are ellipses:

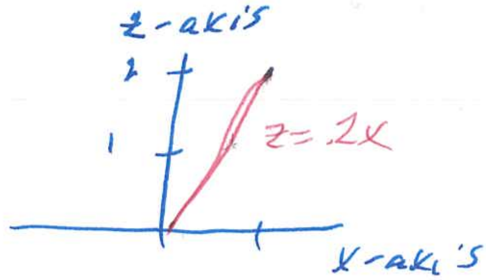


Cross sections are $z=\sqrt{4\cdot 0^2+y^2}$, i.e. $z=y$

and $z=\sqrt{4x^2+0^2}$, i.e. $z=2x$

These are lines





The graph of $z=\sqrt{4x^2+y^2}$ is an elliptic cone