

Calculus 2 Lecture 24

20.09.2018

A. Ram

④

Example 6.9 Solve $y'' + 49y = 28 \sin(7t)$.Since $\sin(7t) = \frac{1}{2i}(e^{7it} - e^{-7it})$ this equation is

$$y'' + 49y = \frac{14}{i}(e^{7it} - e^{-7it})$$

Let $D = \frac{d}{dt}$. Then $y'' + 49y = 0$ is
 $D = (D^2 + 49)y = (D + 7i)(D - 7i)y$ which has solution

$$y_H = c_1 e^{7it} + c_2 e^{-7it}$$

Let $y_p = a t e^{7it} + b t e^{-7it}$ and solve for a and b . Then

$$y_p' = a(t + 7i)e^{7it} + b(t - 7i)e^{-7it}$$

$$y_p'' = a(-49te^{7it} + 7ie^{7it} + 7ie^{7it})$$

$$+ b(-49te^{-7it} - 7ie^{-7it} - 7ie^{-7it})$$

$$= a(-49te^{7it} + 14ie^{7it})$$

$$+ b(-49te^{-7it} - 14ie^{-7it}) \text{ and}$$

$$\frac{14}{i}(e^{7it} - e^{-7it}) = 28 \sin(7t) = y_p'' + 49y_p$$

$$= a(-49te^{7it} + 14ie^{7it}) + 49ate^{7it}$$

$$+ b(-49te^{-7it} - 14ie^{-7it}) + 49bte^{-7it}$$

$$\sum \frac{14}{i} (e^{7it} - e^{-7it}) = a 14i e^{7it} - b 14i e^{-7it}$$

$$\sum a = \frac{14}{i} \frac{1}{14i} = \frac{1}{i^2} = -1 \text{ and } b = \frac{14}{i} \frac{1}{14i} = -1.$$

$\sum y_p = -te^{7it} - te^{-7it}$ and the general solution is

$$y = y_H + y_p = c_1 e^{7it} + c_2 e^{-7it} - te^{7it} - te^{-7it} \quad //$$