Calculus 2 Lecture 15

Integration by parts – i.e. product rule.

Example 4.14 Evaluate $\int x^2 \log x \, dx$.

Solution Since

$$\frac{d}{dx}(x^3 \log x) = 3x^2 \log x + x^3 \frac{1}{x}$$

then

$$\int 3x^2 \log x \, dx = \int \left(\frac{d}{dx}(x^3 \log x) - x^3 \frac{1}{x} \right) dx$$

$$= x^3 \log x - \int x^2 \, dx = x^3 \log x - \frac{1}{3} x^3 + c.$$

So

$$\int x^2 \log x \, dx = \frac{1}{3} x^3 \log x - \frac{1}{9} x^3 + c,$$

where c is a constant.

Example 4.15 Evaluate $\int x e^{5x} dx$.

Solution: Since $\frac{d}{dx}(x e^{5x}) = 5x e^{5x} + e^{5x}$

then

$$\int 5x e^{5x} dx = \int \left(\frac{d}{dx}(x e^{5x}) - e^{5x} \right) dx$$

$$= x e^{5x} - \frac{1}{5} e^{5x} + c.$$

So

$$\int x e^{5x} dx = \frac{1}{5} x e^{5x} - \frac{1}{25} e^{5x} + c,$$

where c is a constant.

Example 4.16 Evaluate $\int \log x \, dx$.

Solution: Since $\frac{d(x\log x)}{dx} = x\frac{1}{x} + \log x$

so $\int \log x \, dx = \int \left(\frac{d(x\log x)}{dx} - x\frac{1}{x}\right) dx$

$= x\log x - \int dx = x\log x - x + c,$

where c is a constant.

Example 4.17 Evaluate $\int e^{3x}\sin(2x)\,dx$ using integration by parts.

Solution: Since

$$\frac{d}{dx}(e^{3x}\sin(2x)) = 3e^{3x}\sin(2x) + e^{3x}\cos(2x)$$

then $\frac{1}{3}\int 3e^{3x}\sin(2x)\,dx = \frac{1}{3}\int\left(\frac{d}{dx}(e^{3x}\sin(2x)) - e^{3x}\cos(2x)\right)dx$

$= \frac{1}{3}e^{3x}\sin(2x) - \frac{1}{3}\int e^{3x}\cos(2x)\,dx.$

Since $\frac{d}{dx}(e^{3x}\cos(2x)) = 3e^{3x}\cos(2x) - e^{3x}\sin 2x$ then

$\frac{1}{3}e^{3x}\sin(2x) - \frac{1}{3}\int e^{3x}\cos 2x\,dx = \frac{1}{3}e^{3x}\sin 2x - \frac{1}{9}\int 3e^{3x}\cos 2x\,dx$

$= \frac{1}{3}e^{3x}\sin 2x - \frac{1}{9}\int\left(\frac{d}{dx}(e^{3x}\cos 2x) + e^{3x}\sin 2x\right)dx$

$$= \frac{1}{3}e^{3x}\sin 2x - \frac{1}{9}e^{3x}\cos 2x - \frac{1}{9}\int e^{3x}\sin 2x\,dx.$$

So

$$\int e^{3x}\sin 2x\,dx = \frac{1}{3}e^{3x}\sin 2x - \frac{1}{9}e^{3x}\cos 2x - \frac{1}{9}\int e^{3x}\sin 2x\,dx$$

So

$$\frac{10}{9}\int e^{3x}\sin 2x\,dx = \frac{1}{3}e^{3x}\sin 2x - \frac{1}{9}e^{3x}\cos 2x + c$$

So

$$\int e^{3x}\sin 2x\,dx = \frac{3}{10}e^{3x}\sin 2x - \frac{1}{10}e^{3x}\cos 2x + c,$$

where c is a constant.