Differential Equations

(18a) Check that $\frac{dy}{dx}=1$ when $y=x+c$, where c is a constant.

If $y=x+c$ then $\frac{dy}{dx}=\frac{d}{dx}(x+c)=1+0=1$.

(18b) Check that $\frac{dy}{dx}=y$ when $y=Ce^x$, where C is a constant.

If $y=Ce^x$ then $\frac{dy}{dx}=\frac{d}{dx}(Ce^x)=Ce^x=y$.

(18c) Check that $\frac{dy}{dx}=\frac{-x}{y}$ when $x^2+y^2=c$, where c is a constant.

If $x^2+y^2=c$ then $2x+2y\frac{dy}{dx}=0$ and

$$\frac{dy}{dx}=\frac{-2x}{2y}=-\frac{x}{y}.$$

(18d) Check that $\frac{dy}{dx}=2y-4x$ when $y=Ce^{2x}+2x+1$, where C is a constant.

If $y=Ce^{2x}+2x+1$ then $\frac{dy}{dx}=2Ce^{2x}+2$ and

$$2y-4x=2(Ce^{2x}+2x+1)-4x=2Ce^{2x}+4x+2-4x$$

$$=2Ce^{2x}+2.$$

So $\frac{dy}{dx} = 2y - 4x$.

(18e) Check that $\frac{dy}{dx} = \frac{-2y}{x}$ when $y = \frac{c}{x^2}$, where c is a constant.

If $y = \frac{c}{x^2}$ then $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{c}{x^2}\right) = \frac{d}{dx}(cx^{-2})$

$$= c(-2)x^{-3} = -\frac{2c}{x^3}.$$

Then $\frac{-2y}{x} = \frac{-2\left(\frac{c}{x^2}\right)}{x} = \frac{-2c}{x^3}$. So $\frac{dy}{dx} = \frac{-2y}{x}$.

(19a) Find solutions of $\frac{dy}{dx} = e^x y$.

If $\frac{dy}{dx} = e^x y$ then $\frac{dy}{y} = e^x dx$ and $\ln y = e^x + c$

so that $y = Ce^{e^x}$, where $C = e^c$ is a constant.

None of these solutions are constant functions of x.

(19b) Find solutions of $\frac{dy}{dx} = xe^y$.

If $\frac{dy}{dx} = xe^y$ then $e^{-y}dy = x\,dx$ and $-e^{-y} = \frac{1}{2}x^2 + c$,

where c is a constant. So $e^{-y} = -\frac{1}{2}x^2 - c$, and $-y = \ln(-\frac{1}{2}x^2+c)$ so $y = -\ln(\frac{-1}{2}x^2+c)$.

(19c) Find solutions of $\frac{dy}{dx} = (\sin x)(y^2+1)$.

If $\frac{dy}{dx} = (\sin x)(y^2+1)$ then $\frac{dy}{y^2+1} = \sin x\, dx$

and $\arctan(y) = -\cos x + c$, where c is a constant.

So $y = \tan(-\cos x + c)$

(19d) Find solutions $\frac{dy}{dx} = (x^2-1)\sin 2y$.

If $\frac{dy}{dx} = (x^2-1)\sin 2y$ then $\frac{dy}{\sin 2y} = (x^2-1)dx$ and

$(\csc 2y)dy = (x^2-1)dx$ So that

(20a) Solve $\frac{dy}{dx}=1$ with $y(0)=3$.

If $\frac{dy}{dx}=1$ then $dy=dx$ so that $y=x+c$,

and if $y(0)=3$ then $3=0+c$ so that $y=x+3$.

(20b) Solve $\frac{dy}{dx}=y$ with $y(0)=2$.

If $\frac{dy}{dx}=y$ then $\frac{dy}{y}=dx$ and $\ln y=x+c$ giving

$y=Ce^x$ where C is a constant. Since

$y(0)=2$ then $2=Ce^0=C$. So $y=2e^x$.

(20c) Solve $\frac{dy}{dx}=\frac{-x}{y}$ with $y(3)=4$.

If $\frac{dy}{dx}=\frac{-x}{y}$ then $y\,dy=-x\,dx$ and $\frac{1}{2}y^2=-\frac{1}{2}x^2+c$.

Since $y(3)=4$ then $\frac{1}{2}4^2=\frac{-1}{2}3^2+c$ giving $8=-\frac{9}{2}+c$

and $c=\frac{25}{2}$. So $\frac{1}{2}y^2+\frac{1}{2}x^2=\frac{25}{2}$ or $x^2+y^2=5^2$.

(20d) Solve $\frac{dy}{dx}=2y-4x$ with $y(-1)=0$.

(20e) Solve $\frac{dy}{dx} = \frac{-2y}{x}$ with $y(2) = 64$.

If $\frac{dy}{dx} = \frac{-2y}{x}$ then $\frac{dy}{y} = -\frac{2}{x}dx$ and $\ln y = -2\ln x + c$

so that $y = Cx^{-2}$ where c is a constant. Then

$64 = y(2) = C\,2^{-2}$ so that $C = 64 \cdot 4 = 4^4$.

So $y = 4^4 \frac{1}{x^2}$.

(21a) Verify that $y = e^{-2x} + e^{3x}$ is a solution of

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$.

Let $y = e^{-2x} + e^{3x}$. Then $\frac{dy}{dx} = -2e^{-2x} + 3e^{3x}$ and

$\frac{d^2y}{dx^2} = 4e^{-2x} + 9e^{3x}$. So

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 4e^{-2x} + 9e^{3x} - (-2e^{-2x} + 3e^{3x}) - 6(e^{-2x} + e^{3x})$$

$$= (4+2-6)e^{-2x} + (9-3-6)e^{3x} = 0.$$

(21b) Verify that $x = C\sin(nt)$ is a solution of

$\frac{d^2x}{dt^2} = -n^2x$.

Let $x = C\sin(nt)$. Then $\frac{dx}{dt} = nC\cos(nt)$ and

$$\frac{d^2x}{dt^2} = -n^2 C\sin(nt) = -n^2 x.$$

(21c) Verify that $y = \frac{4}{x+1}$ is a solution of

$$\frac{d^2y}{dx^2} = \frac{2}{y}\left(\frac{dy}{dx}\right)^2.$$

Let $y = \frac{4}{x+1} = 4(x+1)^{-1}$ Then $\frac{dy}{dx} = -4(x+1)^{-2}$ and

$\frac{d^2y}{dx^2} = 8(x+1)^{-3}$. So

$$\frac{2}{y}\left(\frac{dy}{dx}\right)^2 = \frac{2}{4(x+1)^{-1}}\left(-4(x+1)^{-2}\right)^2 = \frac{2}{4}4^2(x+1)^{-3} = 8(x+1)^{-3}$$

$$= \frac{d^2y}{dx^2}.$$

(22) Find constants a, b, c, d so that $y = ax^3+bx^2+cx+d$ is a solution of $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x^3$.

Let $y = ax^3+bx^2+cx+d$. Then $\frac{dy}{dx} = 3ax^2+2bx+c$

and $\frac{d^2y}{dx^2} = 6ax+2b$. Assume

$$x^3 = \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 6ax+2b+2(3ax^2+2bx+c)$$
$$+ax^3+bx^2+cx+d$$

$$= ax^3+(6a+b)x^2+(6a+4b+c)x+(2b+2c+d)$$

giving $a=1$, $6a+b=0$, $6a+4b+c=0$, $2b+2c+d=0$.

So $a=1$, $\quad c=-6a-4b=-6+24=18$

$b=-6a=-6$, $\quad d=-2b-2c=12-36=-24$.

(23) Find a constant k such that $f(x)=e^{kx}$ is a solution of $f''(x)+3f'(x)+2f(x)=0$.

Let $f=e^{kx}$. Then $f'=ke^{kx}$ and $f''=k^2e^{kx}$.

Assume $f''+3f'+2f=0$. Then

$0=k^2e^{kx}+3ke^{kx}+2e^{kx}=(k^2+3k+2)e^{kx}$

$=(k+2)(k+1)e^{kx}$.

So $k=-2$ or $k=-1$.

(24a) Solve $\frac{dy}{dx}=\frac{1}{\sqrt{x}}$.

If $\frac{dy}{dx}=\frac{1}{\sqrt{x}}$ then $dy=x^{-\frac{1}{2}}dx$ and $y=2x^{\frac{1}{2}}+c$,

where c is a constant.

(24b) Solve $\frac{dy}{dt}=\frac{t^2+3t-1}{t}$.

If $\frac{dy}{dt}=\frac{t^2+3t-1}{t}=t+3-t^{-1}$ then $y=\frac{1}{2}t^2+3t-\ln t+c$,

where c is a constant.

(24c) Solve $\frac{dy}{dt} = \sin(3t+\pi)$ with $y(0)=1$.

If $\frac{dy}{dt} = \sin(3t+\pi)$ then $y = -\frac{1}{3}\cos(3t+\pi) + c$,

where c is a constant.

(24d) Solve $\frac{dy}{dx} = \frac{1}{2x-1}$ with $y(1)=3$.

If $\frac{dy}{dx} = \frac{1}{2x-1} = (2x-1)^{-1}$ then $y = \frac{1}{2}\ln(2x-1) + c$,

where c is a constant.

(25a) Solve $\frac{d^2y}{dx^2} = e^{x/2}$.

If $\frac{d^2y}{dx^2} = e^{x/2}$ then $\frac{dy}{dx} = 2e^{x/2} + c_1$ and $y = 4e^{x/2} + c_1x + c_2$,

where c_1 and c_2 are constants.

(25b) Solve $\frac{d^2y}{dt^2} = \sqrt{1-t}$.

If $\frac{d^2y}{dt^2} = \sqrt{1-t} = (1-t)^{1/2}$ then $\frac{dy}{dt} = -\frac{2}{3}(1-t)^{3/2} + c_1$

and $y = +\frac{2}{3}\cdot\frac{2}{5}(1-t)^{5/2} + c_1t + c_2 = \frac{4}{15}(1-t)^{5/2} + c_1t + c_2$,

where c_1 and c_2 are constants.

(25c) Solve $\frac{d^2y}{dx^2}=\frac{1}{(x+1)^2}$.

If $\frac{d^2y}{dx^2}=\frac{1}{(x+1)^2}=(x+1)^{-2}$ then $\frac{dy}{dx}=-(x+1)^{-1}+c_1$

and $y=-\ln(x+1)+c_1x+c_2$, where c_1 and c_2 are constants.

(26a) Solve $\frac{dy}{dx}=\frac{1}{y^2}$.

If $\frac{dy}{dx}=\frac{1}{y^2}$ then $y^2\frac{dy}{dx}=1$ and $\int y^2\frac{dy}{dx}\,dx=\int dx$

so that $\frac{1}{3}y^3=x+c$, where c is a constant.

(26b) Solve $\frac{dy}{dx}=1+y^2$

If $\frac{dy}{dx}=1+y^2$ then $\frac{1}{1+y^2}\frac{dy}{dx}=1$ and

$\int\frac{1}{1+y^2}\frac{dy}{dx}\,dx=\int dx$ so that $\arctan(y)=x+c$,

where c is a constant. So $y=\tan(x+c)$.

(26c) Solve $\frac{dy}{dx} = \sqrt{y}$ with $y(3) = 1$.

If $\frac{dy}{dx} = \sqrt{y}$ then $\frac{1}{\sqrt{y}} \frac{dy}{dx} = 1$ so $\int y^{-\frac{1}{2}} \frac{dy}{dx}\, dx = \int dx$

and $2y^{1/2} = x + c$ where c is a constant.

Since $y(3) = 1$ then $2 \cdot 3^{1/2} = 1 + c$ and $c = 2\sqrt{3} - 1$.

So $2y^{\frac{1}{2}} = x + 2\sqrt{3} - 1$ and $y = \left(\frac{x + 2\sqrt{3} - 1}{2}\right)^2$

(26d) Solve $\frac{dy}{dx} = y - 4$ with $y(0) = 5$.

If $\frac{dy}{dx} = y - 4$ then $\frac{1}{y-4} \frac{dy}{dx} = 1$ and $\int \frac{1}{y-4} \frac{dy}{dx}\, dx = \int dx$

so that $\ln(y-4) = x + c$. So $y - 4 = Ce^x$, where

C is a constant. Since $y(0) = 5$ then

$5 - 4 = Ce^0 = C$ and $C = 1$. So $y = e^x + 4$.

(27a) Solve $\frac{dy}{dx} = 5y^2 \cos x$.

If $\frac{dy}{dx} = 5y^2 \cos x$ then $\frac{1}{5} y^{-2} \frac{dy}{dx} = \cos x$ and

$\int \frac{1}{5} y^{-2} \frac{dy}{dx} dx = \int \cos x \, dx$ giving $\frac{1}{5} \cdot \frac{1}{-1} y^{-1} = \sin x + c$,

where c is a constant. So $y = \frac{-5}{\sin x + C}$.

(27b) Solve $\frac{dy}{dx} = e^x e^{-2y}$

If $\frac{dy}{dx} = e^x e^{-2y}$ then $e^{2y} \frac{dy}{dx} = e^x$ and

$\int e^{2y} \frac{dy}{dx} dx = \int e^x dx$ giving $\frac{1}{2} e^{2y} = e^x + c$,

where c is a constant.

(27c) Solve $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{x}$

If $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{x}$ then $\frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = \frac{1}{x}$ and

$\int \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} dx = \int \frac{1}{x} dx$ giving $\arcsin y = \ln x + c$.

So $y = \sin(\ln x + c)$, where c is a constant.

(27d) Solve $\frac{dy}{dt} = 3\sqrt{9-y^2}\,\sin^4 t \cos t$.

If $\frac{dy}{dt} = 3\sqrt{9-y^2}\,\sin^4 t \cos t$ then $\frac{1}{9-y^2}\frac{dy}{dt} = 3\sin^4 t \cos t$.

So $\int \frac{1}{3\sqrt{1-\left(\frac{y}{3}\right)^2}}\frac{dy}{dt}\,dt = \int 3\sin^4 t\cos t\,dt$ giving

$\frac{1}{3}\arcsin\left(\frac{y}{3}\right)\cdot 3 = 3\,\frac{1}{5}\sin^5 t + \tilde{c}$ so that

$y = \sin\left(\frac{3}{5}\sin^5 t + c\right)$ where c is a constant.

(27e) Solve $\frac{dx}{dt} = \frac{3t+e^{2t}}{x^2+e^{-x}}$.

If $\frac{dx}{dt} = \frac{3t+e^{2t}}{x^2+e^{-x}}$ then $(x^2+e^{-x})\frac{dx}{dt} = 3t+e^{2t}$.

So $\int (x^2+e^{-x})\frac{dx}{dt}\,dt = \int (3t+e^{2t})\,dt$.

So $\frac{1}{3}x^3 + (-1)e^{-x} = \frac{3}{2}t^2 + \frac{1}{2}e^{2t} + c$;

So $\frac{1}{3}x^3 - e^{-x} = \frac{3}{2}t^2 + \frac{1}{2}e^{2t} + c$, where c is a constant.

(27f) Solve $\frac{dy}{dt} = \frac{(y^2+1)\cos^2 3t}{2y}$

If $\frac{dy}{dt} = \frac{(y^2+1)\cos^2 3t}{2y}$ then $\frac{2y}{y^2+1}\frac{dy}{dt} = \cos^2 3t$.

So $\int \frac{2y}{y^2+1}\frac{dy}{dt}\,dt = \int \cos^2 3t\,dt = \int \left(\frac{e^{i3t}+e^{-i3t}}{2}\right)^2 dt$

$$= \int \frac{e^{i6t}+2+e^{-i6t}}{4}\,dt = \frac{1}{4}\left(\frac{e^{i6t}}{6i} + 2t + \frac{e^{-i6t}}{6i}\right) + c$$

$$= \frac{1}{4}\cdot\frac{1}{3}\sin 6t + \frac{t}{2} + c = \frac{1}{12}\sin 6t + \frac{t}{2} + c,$$

where c is a constant.

So $\ln(y^2+1) = \frac{1}{12}\sin 6t + \frac{t}{2} + c$, where c is a constant.

(28a) Solve $\frac{dy}{dx} = 3xy$ with $y(0)=3$.

If $\frac{dy}{dx} = 3xy$ then $\frac{1}{y}\frac{dy}{dx} = 3x$ and

$\int \frac{1}{y}\frac{dy}{dx}\,dx = \int 3x\,dx$ giving $\ln y = \frac{3}{2}x^2 + c$, where c is a constant. So $y = Ce^{3/2 x^2}$, where $C = e^c$ is a constant. Since $y(0)=3$ then $3 = Ce^{3/2\cdot 0} = C$.

So $y = 3e^{3/2 x^2}$.

(28b) Solve $\frac{dx}{dt} = \frac{x}{t(t+1)}$ with $x(1)=1$.

If $\frac{dx}{dt} = \frac{x}{t(t+1)}$ then $\frac{1}{x}\frac{dx}{dt} = \frac{1}{t(t+1)} dt$

So $\int \frac{1}{x}\frac{dx}{dt}\,dt = \int \frac{1}{t(t+1)}\,dt = \int \left(\frac{1}{t} + \frac{-1}{t+1}\right) dt$

$$= \ln t - \ln(t+1) + c = \ln\left(\frac{t}{t+1}\right) + c.$$

So $\ln x = \ln\left(\frac{t}{t+1}\right) + c$ and $x = C\frac{t}{t+1}$, where

$C = e^c$ is a constant.

Since $x(1)=1$ then $1 = C\frac{1}{1+1} = C\frac{1}{2}$ and $C=2$.

So $x = \frac{2t}{t+1}$.

(29) The height of a projectile fired vertically upwards from the ground at an initial speed of 50m/s satisfies $\frac{d^2h}{dt^2} = -10$.

(a) The initial conditions are $h(0)=0$ and $h'(0)=50$.

(b) Since $\frac{d^2h}{dt^2} = -10$ then $\frac{dh}{dt} = -10t + c_1$, where c_1 is a constant. Since $h'(0)=50$ then

$50 = -10 \cdot 0 + c_1$ and $c_1 = 50$.

Since $\frac{dh}{dt} = -10t + 50$, then $h = -5t^2 + 50t + c_2$.

Since $h(0) = 0$ then $0 = -5 \cdot 0^2 + 50 \cdot 0 + c_2$,

giving $c_2 = 0$. So $h(t) = -5t^2 + 50t$.

The maximum height occurs when $h'(t) = 0$.
If $h'(t) = 0$ then $0 = -10t + 50$ and $t = 5$.

So the maximum height is

$h(5) = -5 \cdot 5^2 + 50 \cdot 5 = -125 + 250 = 125\,m$.

(30) The volume V of a balloon increases at a rate inversely proportional to the current volume.

(a) Write a differential equation satisfied by the volume.

$\frac{dV}{dt}$ = change in volume = (constant) $\frac{1}{V} = \frac{\alpha}{V}$, where α is a constant.

(b) Assume the initial volume is $10\,cm^3$ and after 5 seconds is $40\,cm^3$. Find V.

Since $\frac{dV}{dt} = \frac{\alpha}{V}$ then $V\frac{dV}{dt} = \alpha$ and

$\int V\frac{dV}{dt}dt = \int \alpha \, dt = \alpha t + c$, where c is a constant.

So $\frac{1}{2}V^2 = \alpha t + c$ and $V(0) = 10$ and $V(5) = 40$.

So $\frac{1}{2}10^2 = \alpha \cdot 0 + c$ and $\frac{1}{2}\cdot 40^2 = \alpha \cdot 5 + c$.

So $c = 50$ and $800 = 5\alpha + 50$, and $\alpha = \frac{750}{5} = 150$.

So $\frac{1}{2}V^2 = 150t + 50$. So $V = \sqrt{300t + 100}$.

1(c) What is the volume after 8 seconds.

$V(8) = \sqrt{300\cdot 8 + 100} = \sqrt{2500} = 50 \text{ cm}^3$.

(31) The rate of decay of amount Q is proportional to current amount.

(a) Write a differential equation for the amount Q.

$\frac{dQ}{dt} = \alpha Q$ where α is a constant.

(b) Solve the differential equation.

Since $\frac{dQ}{dt} = \alpha Q$ then $\frac{1}{Q}\frac{dQ}{dt} = \alpha$ and

$\int \frac{1}{Q}\frac{dQ}{dt}dt = \int \alpha \, dt = \alpha t + c$, where c is a constant.

So $\ln Q = \alpha t + c$ and $Q = Ce^{\alpha t}$, where $C = e^c$ is a constant.

(b) If the initial amount is 100 and $Q=50$ when $t=10$ then find Q.

Since $Q=Ce^{\alpha t}$ and $Q(0)=100$ and $Q(10)=50$,

then $100=Ce^{\alpha\cdot 0}=C$ and

$50=Q(10)=Ce^{\alpha 10}=100e^{\alpha 10}$ so $10\alpha=\ln\frac{50}{100}=\ln\frac{1}{2}$

and $\alpha=\frac{1}{10}\ln\frac{1}{2}=\frac{-1}{10}\ln 2$.

So $Q=100e^{-\frac{1}{10}(\ln 2)t}$.

(32) The number of fish in a lake satisfies

$\frac{dF}{dt}=0.1F$.

(a) Find F if the initial number of fish is 10.

Since $\frac{dF}{dt}=0.1F$ then $\frac{1}{F}\frac{dF}{dt}=0.1$ and

$\int\frac{1}{F}\frac{dF}{dt}dt=\int 0.1dt$ giving $\ln F=0.1t+c$

and $F=Ce^{0.1t}$, where c and $C=e^{c}$ are constants.

Since $F(0)=10$ then $10=Ce^{0.1\cdot 0}=C$ and

$F=10e^{0.1t}$.

(b) When is F equal to 1000.

If $1000 = F = 10e^{0.1t}$ then $0.1t = \ln \frac{1000}{10} = \ln 100$.

So $t = 10 \ln 100$.

(33) The population P of moose decreases proportional to the current population.
Write a differential equation for the population.

$\frac{dP}{dt} = -\alpha P$, where α is a constant.

(b) Solve for P.

Since $\frac{dP}{dt} = -\alpha P$ then $\frac{1}{P}\frac{dP}{dt} = -\alpha$ and

$\int \frac{1}{P}\frac{dP}{dt}\,dt = \int -\alpha\,dt$ giving $\ln P = -\alpha t + c$ and

$P = Ce^{-\alpha t}$, where c and $C = e^{c}$ are constants.

(c) Determine P given that the initial value is 100 and after 2 years P is 110.

Since $100 = Ce^{-\alpha \cdot 0} = C$ and $110 = Ce^{-\alpha \cdot 2} = 100e^{-2\alpha}$

then $-2\alpha = \ln \frac{110}{100}$ and $\alpha = -\frac{1}{2}\ln 1.1$.

So $P = 100e^{(\frac{1}{2}\ln 1.1)t}$.

(d) Find the population after 5 years.

$$P(5) = 100\, e^{\left(\frac{1}{2} \ln 1.1\right) 5} = 100\, e^{5/2 \ln 1.1}.$$

Calculus 1 Topic 4 Newton's law of cooling

(34) A 110°C metal rod is placed into a 10°C cooling tank. After 2 minutes the temperature of the rod is 70°C. The cooling law is

$$\frac{dT}{dt} = -k(T - T_s), \text{ where } T_s = 10.$$

(a) Find the temperature of the rod.

Since $\frac{dT}{dt} = -k(T-10)$ then $\frac{1}{(T-10)}\frac{dT}{dt} = -k$ and

so $\int \frac{1}{T-10} \cdot \frac{dT}{dt}\, dt = \int -k\, dt$ giving $\ln(T-10) = -kt + c$,

and $T - 10 = C e^{-kt}$, where c and $C = e^c$ are constants.

So $T = C e^{-kt} + 10$.

Since $110 = T(0) = C e^0 + 10 = C + 10$ then $C = 100$

Since $70 = T(2) = C e^{-2k} + 10 = 100 e^{-2k} + 10$ then

$\ln \frac{60}{100} = -2k$ and $k = -\frac{1}{2} \ln\left(\frac{3}{5}\right) = \frac{1}{2} \ln\left(\frac{5}{3}\right)$.

So $T = 100 e^{-\frac{1}{2}\ln\left(\frac{5}{3}\right)t} + 10$.

(b) Find the temperature after a further 2 min.

$$T(t+2) = 100e^{-\frac{1}{2}\ln\left(\frac{5}{3}\right)(t+2)} + 10 = e^{-\ln\left(\frac{5}{3}\right)} \cdot 100e^{-\frac{1}{2}\ln\left(\frac{5}{3}\right)t} + 10$$

$$= e^{-\ln\left(\frac{5}{3}\right)} T(t) + \left(10 - 10e^{-\ln\left(\frac{5}{3}\right)}\right).$$