Example 1.82 Sketch $\{z \in \mathbb{C} \mid z^4+z^2-12=0\}$

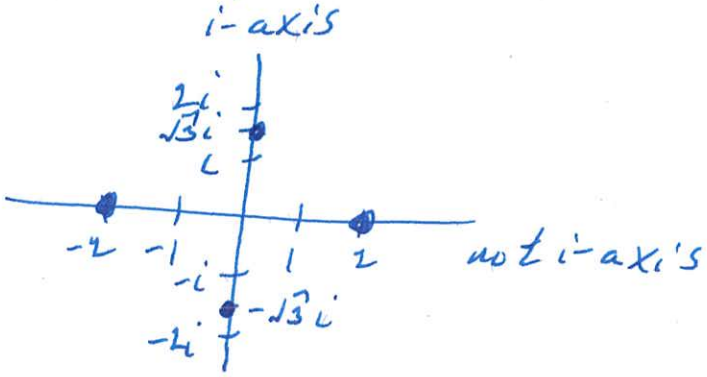
Solution: Since $z^4+z^2-12=(z^2+4)(z^2-3)$ then

$\{z \in \mathbb{C} \mid z^4+z^2-12=0\} = \{z \in \mathbb{C} \mid (z^2+4)(z^2-3)=0\}$

$= \{z \in \mathbb{C} \mid z^2=4 \text{ or } z^2=-3\}$

$= \{z \in \mathbb{C} \mid z=2 \text{ or } z=-2 \text{ or } z=\sqrt{3}i \text{ or } z=-\sqrt{3}i\}$

$= \{2,-2,\sqrt{3}i,-\sqrt{3}i\}$



Example 1.84 Express $q(z)=z^4+z^2-12$ as a product of linear and quadratic factors with all coefficients real.

Solution: $z^4+z^2-12=(z^2+4)(z^2-3)$ //.

Example 1.86 Factorise $p(z)=z^3+z^2+z+1$ over the reals.

Solution: $(z-1)p(z)=(z-1)(z^3+z^2+z+1)$

$= z^4+z^3+z^2+z$

$\quad -z^3-z^2-z-1 = z^4-1.$

So

$$p(z) = z^3 + z^2 + z + 1 = \frac{z^4 - 1}{z - 1} = \frac{(z+1)(z-1)(z+i)(z-i)}{z-1}$$

$$= (z+1)(z+i)(z-i) = (z+1)(z^2+1)$$