<u>MAST 10005</u> Slide
A. Ram 22.07.2019

# The order on $\mathbb{R}$

(a) If $x \in \mathbb{R}$ then $x \leq x$

(b) If $x, y, z \in \mathbb{R}$ and $x \leq y$ and $y \leq z$ then $x \leq z$

(c) If $x, y \in \mathbb{R}$ and $x \leq y$ and $y \leq x$ then $x = y$

(d) If $x, y \in \mathbb{R}$ then $x \leq y$ or $y \leq x$

(e) If $x, y, c \in \mathbb{R}$ ~~then~~ and $x \leq y$ then $x + c \leq y + c$

(f) If $x, y \in \mathbb{R}$ and $x \geq 0$ and $y \geq 0$ then $xy \geq 0$.

$\mathbb{R}$ is an <u>ordered</u> <u>field</u>.

There is no possible order on $\mathbb{C}$ that makes $\mathbb{C}$ into an ordered field.

[LHSI 100U]
A. Rem. 22.07.2019

**Often used:**

(*) If $x, y, a \in \mathbb{R}$ and $x < y$ and $a > 0$ then $ax < ay$

(**) If $x, y, a \in \mathbb{R}$ and $x < y$ and $a < 0$ then $ax > ay$

Proofs on the following page.

Page 59 Show that if $a, x, y \in \mathbb{F}$ and $x < y$ and $a > 0$ then $ax < ay$.

Proof: Assume $a, x, y \in \mathbb{F}$ and $a > 0$ and $x < y$.

Since $x < y$ then $x + (-x) < y + (-x)$.

So $0 < y - x$.

So $a(y-x) > 0$.

So $ay - ax > 0$

So $ay - ax + ax > 0 + ax$

So $ay > ax$ //.

Page 60 Show that if $a, x, y \in \mathbb{F}$ and $a < 0$ and $x < y$ than $ax > ay$.

Proof Assume $a, x, y \in \mathbb{F}$ and $a < 0$.

Then $a + (-a) < 0 + (-a)$.

So $0 < -a$.

So, by page 59, $(-a)x < (-a)y$.

So $-ax < -ay$

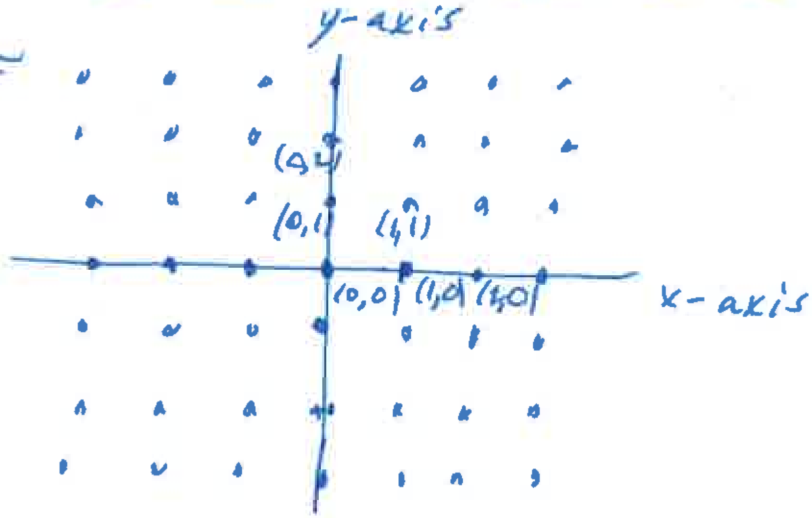
So $-ax + (ax + ay) < -ay + (ax + ay)$

So $ay < ax$. //.

Calculus I

**Example 1.17** Sketch the Cartesian product $\mathbb{Z}\times\mathbb{Z}=\mathbb{Z}^2$ as a subset of $\mathbb{R}^2$

**Solution**




**Example 1.18** Express the set $A=\{x\in\mathbb{R} \mid -2-\frac{1}{2}x>-4\}$ as an interval.

**Solution**

$$A=\{x\in\mathbb{R} \mid -2-\tfrac{1}{2}x>-4\}$$

$$=\{x\in\mathbb{R} \mid -\tfrac{1}{2}x>-2\}=\{x\in\mathbb{R} \mid -x>-4\}$$

$$=\{x\in\mathbb{R} \mid 0>-4+x\}=\{x\in\mathbb{R} \mid 4>x\}$$

$$=\mathbb{R}_{(4,\infty)}.$$

**Example 1.19** Express $A=\{x\in\mathbb{R} \mid 1-x<3x+2\}$ as an interval.

**Solution** $A=\{x\in\mathbb{R} \mid 1-x<3x+2\}$

$$=\{x\in\mathbb{R} \mid 1-x+x<3x+2+x\}=\{x\in\mathbb{R} \mid 1<4x+2\}$$

$$=\{x\in\mathbb{R} \mid -1<4x\}=\{x\in\mathbb{R} \mid -\tfrac{1}{4}<x\}=\mathbb{R}_{(-\frac{1}{4},\infty)}.$$

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Calculus I

**Example 1.22** Prove that if $a, b, x, y \in \mathbb{R}$ and if $x < y$ and $a < b$ then $x + a < y + b$.

**Solution** Assume $a, b, x, y \in \mathbb{R}$ and $x < y$ and $a < b$.

To show: $x + a < y + b$.

$$\begin{aligned} x + a &< y + a, \quad \text{since } x < y \\ &< y + b, \quad \text{since } a < b. \end{aligned}$$

So $x + a < y + b$. //

**Example 1.24** Express $A = \{x \in \mathbb{R} \mid 2e^{5x} - 1 < 5\}$ as an interval.

**Solution**
$$\begin{aligned} A &= \{x \in \mathbb{R} \mid 2e^{5x} - 1 < 5\} \\ &= \{x \in \mathbb{R} \mid 2e^{5x} < 6\} \\ &= \{x \in \mathbb{R} \mid e^{5x} < 3\} \\ &= \{x \in \mathbb{R} \mid 5x < \log 3\} \qquad \left(\begin{array}{l}\text{if } a < b \\ \text{then } \log a < \log b\end{array}\right) \\ &= \{x \in \mathbb{R} \mid x < \tfrac{1}{5} \log 3\} \\ &= \mathbb{R}_{(-\infty, \frac{1}{5}\log 3)}. \; // \end{aligned}$$

Example 1.25

(a) Express $A$ as a union of intervals where

$$A = \{x \in \mathbb{R} \mid (x+1)(x-2) > 0\}$$

Solution $A = \{x \in \mathbb{R} \mid (x+1)(x-2) > 0\}$

$$= \left\{x \in \mathbb{R} \;\middle|\; \begin{array}{l}(x+1>0 \text{ and } x-2>0) \text{ or} \\ (x+1)<0 \text{ and } (x-2)<0\end{array}\right\}$$

$$= \{x \in \mathbb{R} \mid x+1>0 \text{ and } x-2>0\} \cup \left\{x \in \mathbb{R} \;\middle|\; \begin{array}{l}x+1<0 \text{ and} \\ x-2<0\end{array}\right\}$$

$$= \{x \in \mathbb{R} \mid x>-1 \text{ and } x>2\} \cup \{x \in \mathbb{R} \mid x<-1 \text{ and } x<2\}$$

$$= \mathbb{R}_{(2,\infty)} \cup \mathbb{R}_{(-\infty,-1)}.$$

(b) Express $B = \{x \in \mathbb{R} \mid (x+1)(x-2) < 0\}$ is an interval.

Solution: $B = \{x \in \mathbb{R} \mid (x+1)(x-2) < 0\}$

$$= \{x \in \mathbb{R} \mid ((x+1)<0 \text{ and } (x-2)>0) \text{ or } ((x+1)>0 \text{ and } (x-2)<0)\}$$

$$= \{x \in \mathbb{R} \mid x<-1 \text{ and } x>2\} \cup \{x \in \mathbb{R} \mid x>-1 \text{ and } x<2\}$$

$$= \emptyset \cup \mathbb{R}_{(-1,2)} = \mathbb{R}_{(-1,2)} \text{ //}$$

Calculus 1

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Example 1.26 Express $A = \{x \in \mathbb{R} \setminus \{3\} \mid f(x) < 1\}$ as a union of intervals where $f(x) = \dfrac{x^2-5}{x-3}$.

Solution

$$A = \{x \in \mathbb{R} \setminus \{3\} \mid f(x) < 1\}$$

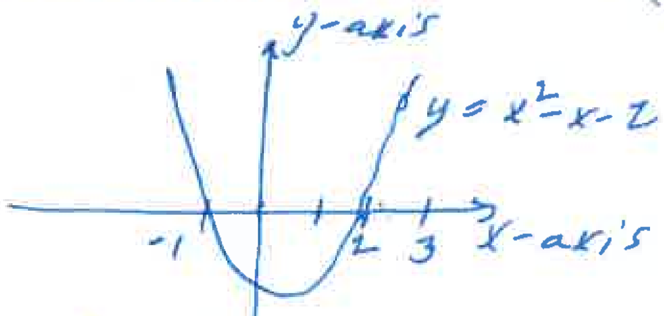
$$= \left\{x \in \mathbb{R} \;\middle|\; x \neq 3 \text{ and } \frac{x^2-5}{x-3} < 1\right\}$$

$$= \left\{x \in \mathbb{R} \;\middle|\; \left(x < 3 \text{ and } \frac{x^2-5}{x-3} < 1\right) \text{ or } \left(x > 3 \text{ and } \frac{x^2-5}{x-3} < 1\right)\right\}$$

$$= \left\{x \in \mathbb{R} \;\middle|\; \begin{array}{l} x-3<0 \text{ and} \\ x^2-5 > x-3 \end{array}\right\} \cup \left\{x \in \mathbb{R} \;\middle|\; \begin{array}{l} x-3>0 \\ \text{and } x^2-5 < x-3 \end{array}\right\}$$

$$= \left\{x \in \mathbb{R} \;\middle|\; \begin{array}{l} x < 3 \text{ and} \\ x^2-x-2 > 0 \end{array}\right\} \cup \left\{x \in \mathbb{R} \mid x > 3 \text{ and } x^2-x-2 < 0\right\}$$

$$= \left\{x \in \mathbb{R} \;\middle|\; \begin{array}{l} x < 3 \text{ and} \\ (x-2)(x+1) > 0 \end{array}\right\} \cup \left\{x \in \mathbb{R} \;\middle|\; \begin{array}{l} x > 3 \text{ and} \\ (x-2)(x+1) < 0 \end{array}\right\}$$




$$= \left\{x \in \mathbb{R} \;\middle|\; \begin{array}{l} x < 3 \text{ and } x > 2 \\ \text{or} \\ x < 3 \text{ and } x < -1 \end{array}\right\} \cup \left\{x \in \mathbb{R} \mid x > 3\right\}$$

$$= \mathbb{R}_{(-\infty,-1)} \cup \mathbb{R}_{(2,3)} \cup \mathbb{R}_{(3,\infty)}.$$